POLICIES IN RELATIONAL CONTRACTS

Daniel Barron & Michael Powell

Kellogg School of Management

MANAGERS MOTIVATE...

Managers motivate agents in long-term relationships

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- Promotions (Benson, Li, and Shue, 2016)
- Hiring decisions (Ariely, Belenzon, and Tsolmon, 2013)
- Capital allocation decisions (Graham, Harvey, and Puri, 2015)

MANAGERS MOTIVATE... AND MANAGE

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But why not settle up with cash? (Baker, Jensen, Murphy, 1988)

DUAL ROLE FOR POLICIES

Relational contracting: future surplus determines feasible incentives

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Policies (history-contingent decision rules) determine:

1. Future surplus produced by each agent
2. What rewards are credible today

Optimal policies may bias decisions to make stronger incentives credible

IMPORTANCE OF BILATERAL SURPLUS

Key feature: agents unable to coordinate on punishing the principal

AGENTS UNABLE TO COORDINATE PUNISHMENT

BRIDGESTONE

Firestone

UNITED RUBBER, CORK, LINOLEUM AND PLASTIC WORKERS OF AMERICA

UNITED FOR FREEDOM

ORG'D SEPT. 12, 1935

Strikes in only 3 of 5 plants (Krueger, Mas, 2004)

IMPORTANCE OF BILATERAL SURPLUS

Key feature: agents unable to coordinate on punishing the principal

Key assumption: each agent observes own relationship with principal, and agents do not communicate with each other

AGENDA

- Illustrative Example
- The General Model
- Main Results
- Applications
- The Role of Private Monitoring

EXAMPLE ILLUSTRATING MECHANISM

One principal, two agents – risk-neutral, deep pockets, discount $\delta < 1$

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In first period…

1. Principal and each agent exchange wage payments $w_{i,t} \in \mathbb{R}$
2. Agent i privately chooses effort $e_{i,t} \in \{0,1\}$ at cost $ce_{i,t}$
3. From i, principal earns output $y_{i,t} \in \{0, H_i\}$, $\Pr[H_i] = pe_{i,t} < 1$
4. Principal and each agent exchange bonus payments $\tau_{i,t} \in \mathbb{R}$

Example Illustrating Mechanism

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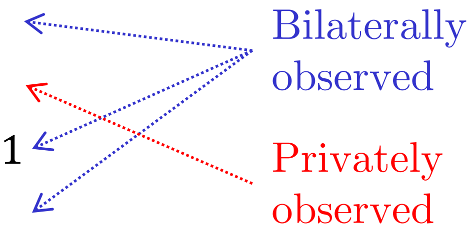

Bilaterally observed

Privately observed

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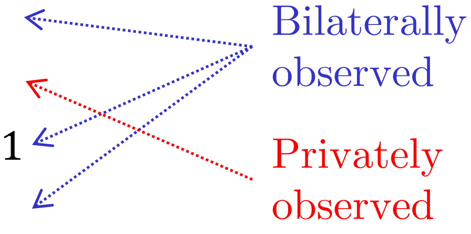

Bilaterally observed

Privately observed

At beginning of second period...

- Principal chooses one of the agents (agent i with probability q_i)
- Plays game repeatedly with chosen agent (other agent produces 0 output)

PAYOFFS

$$\pi = (1 - \delta) \sum_{i=1}^{2} (y_{i,t} - w_{i,t} - \tau_{i,t})$$

$$u_i = (1 - \delta)(w_{i,t} + \tau_{i,t} - ce_{i,t})$$

Assume chosen agent exerts $e_{i,t} = 1$ from second period onwards

MOTIVATING EACH AGENT

What motivates agent i in first period? Following output vector y,

$$B_i(y) = (1-\delta)\tau_{i,t} + \delta U_{i,t}$$

Agent i works hard if:

$$E[B_i(H_i, y_{-i,t})\,] - E[B_i(0, y_{-i,t})] \geq (1-\delta)\frac{c}{p}$$

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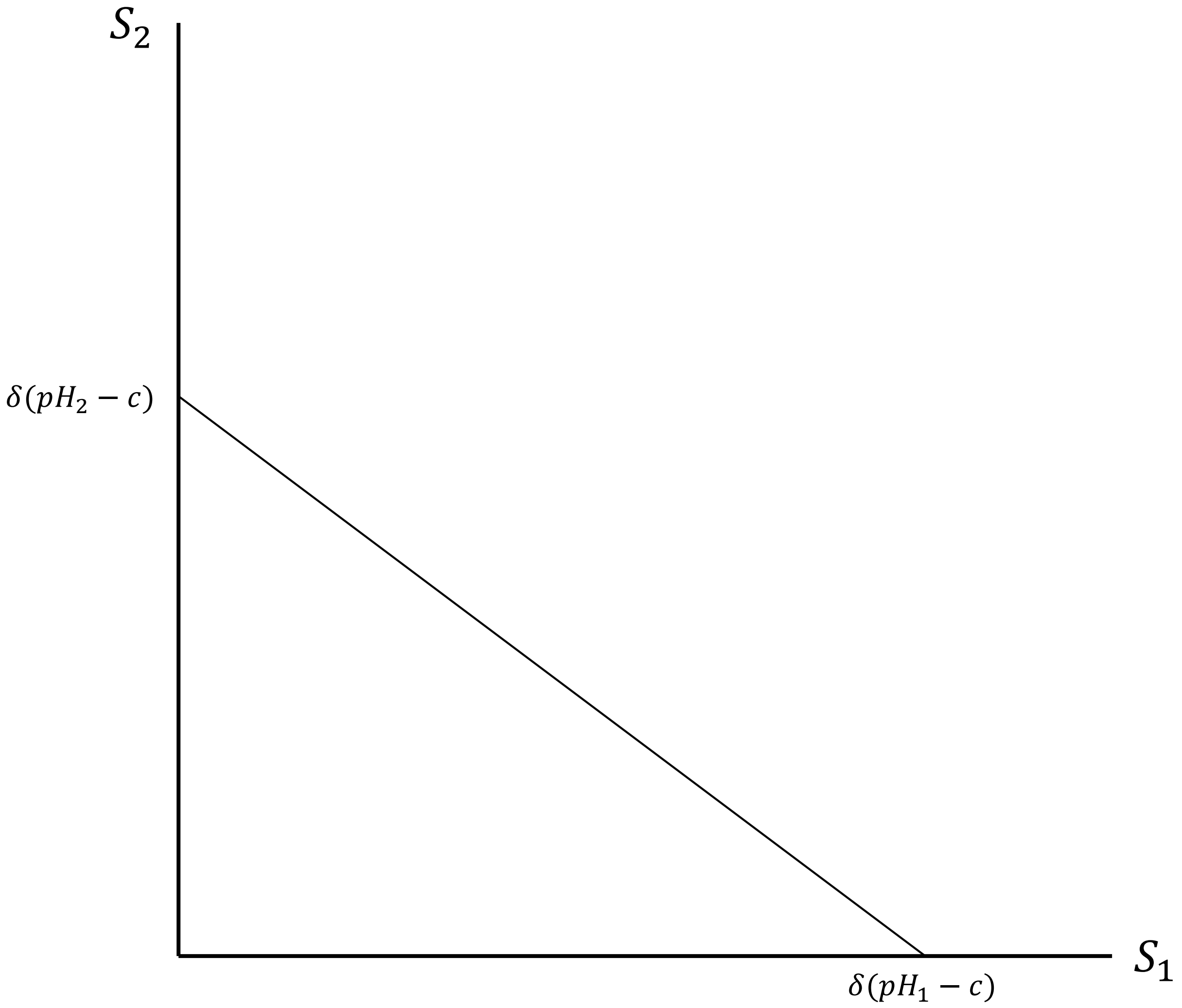
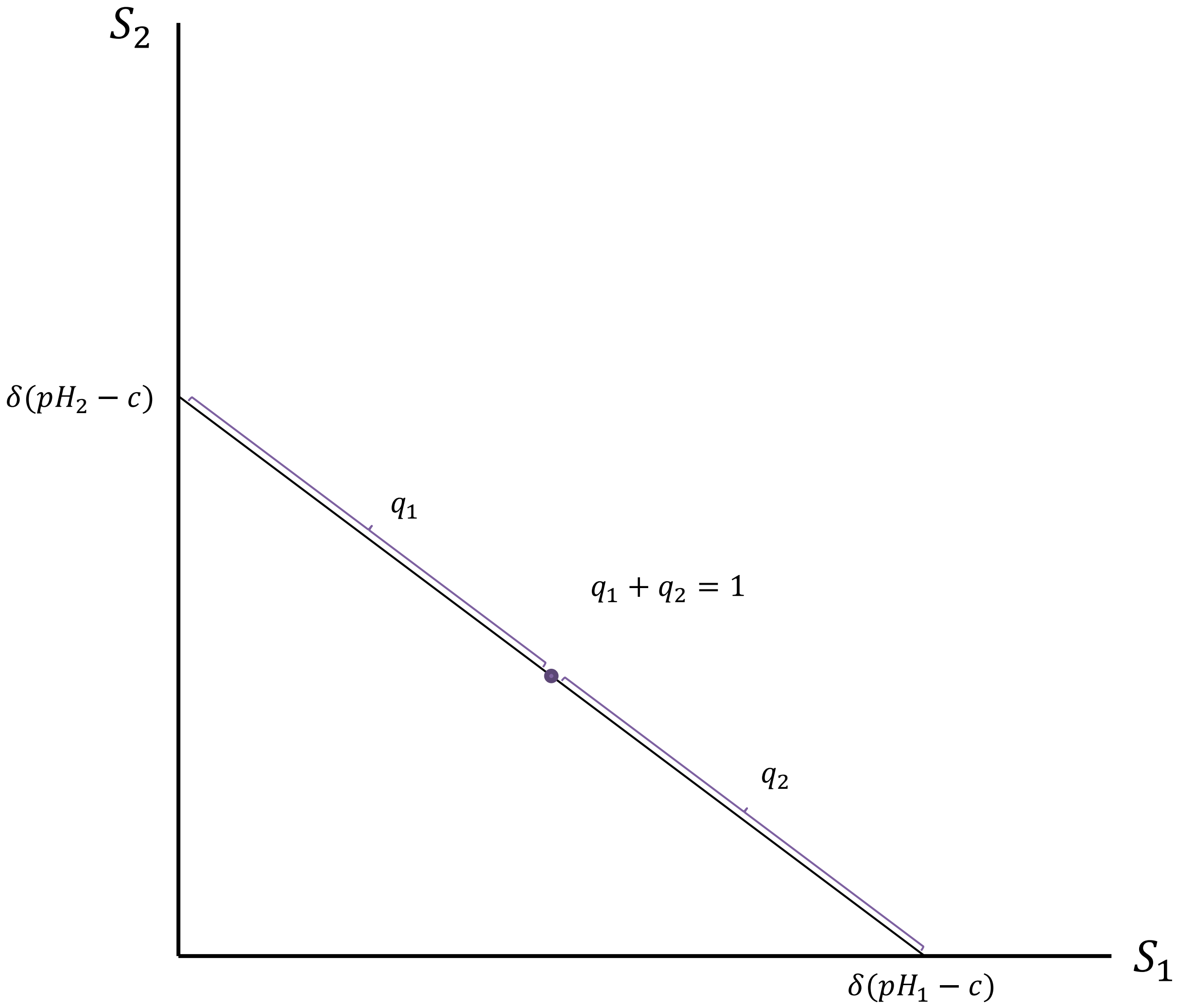
Necessary and also sufficient, in a particular sense.

Dynamic Enforcement Constraint



DYNAMIC ENFORCEMENT CONSTRAINT



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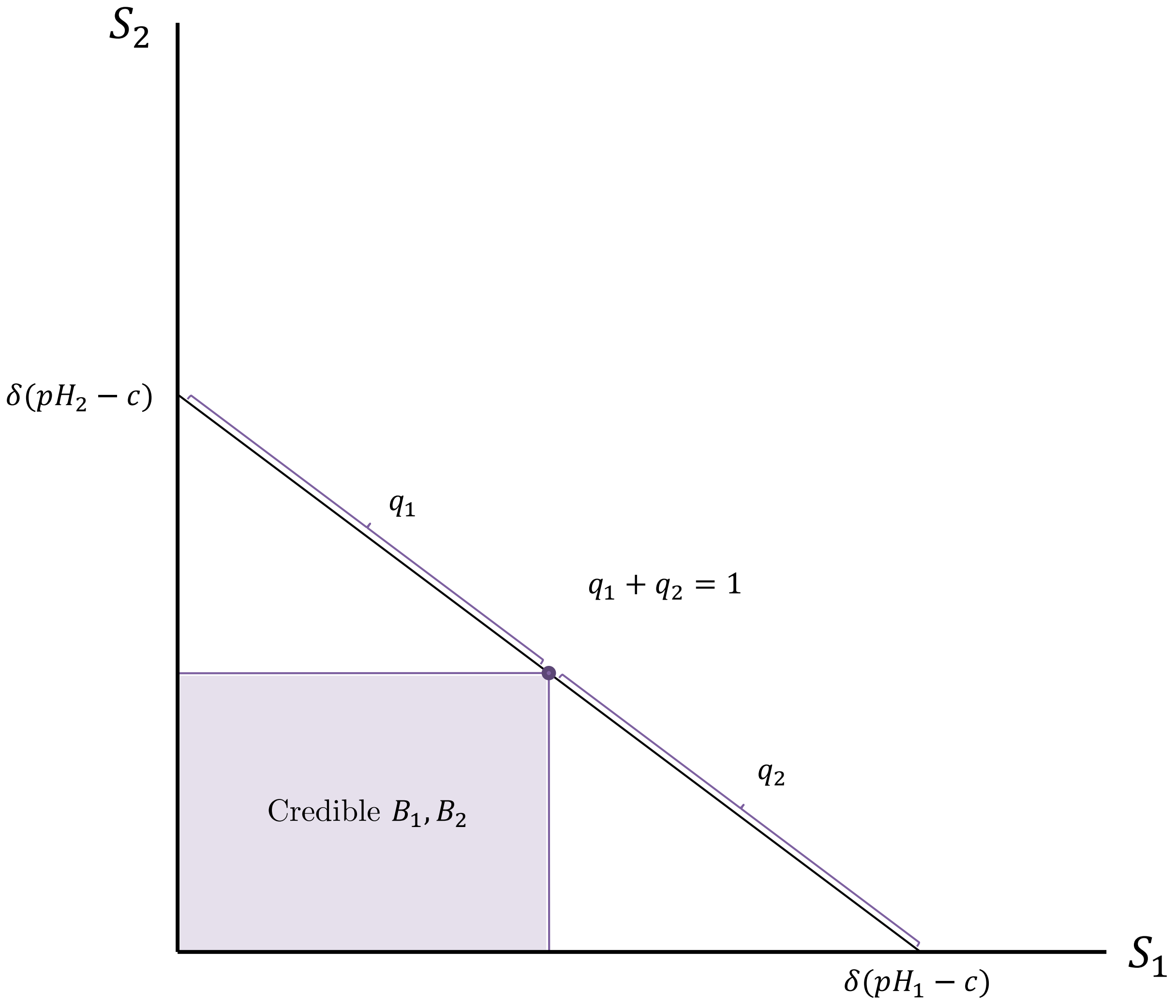
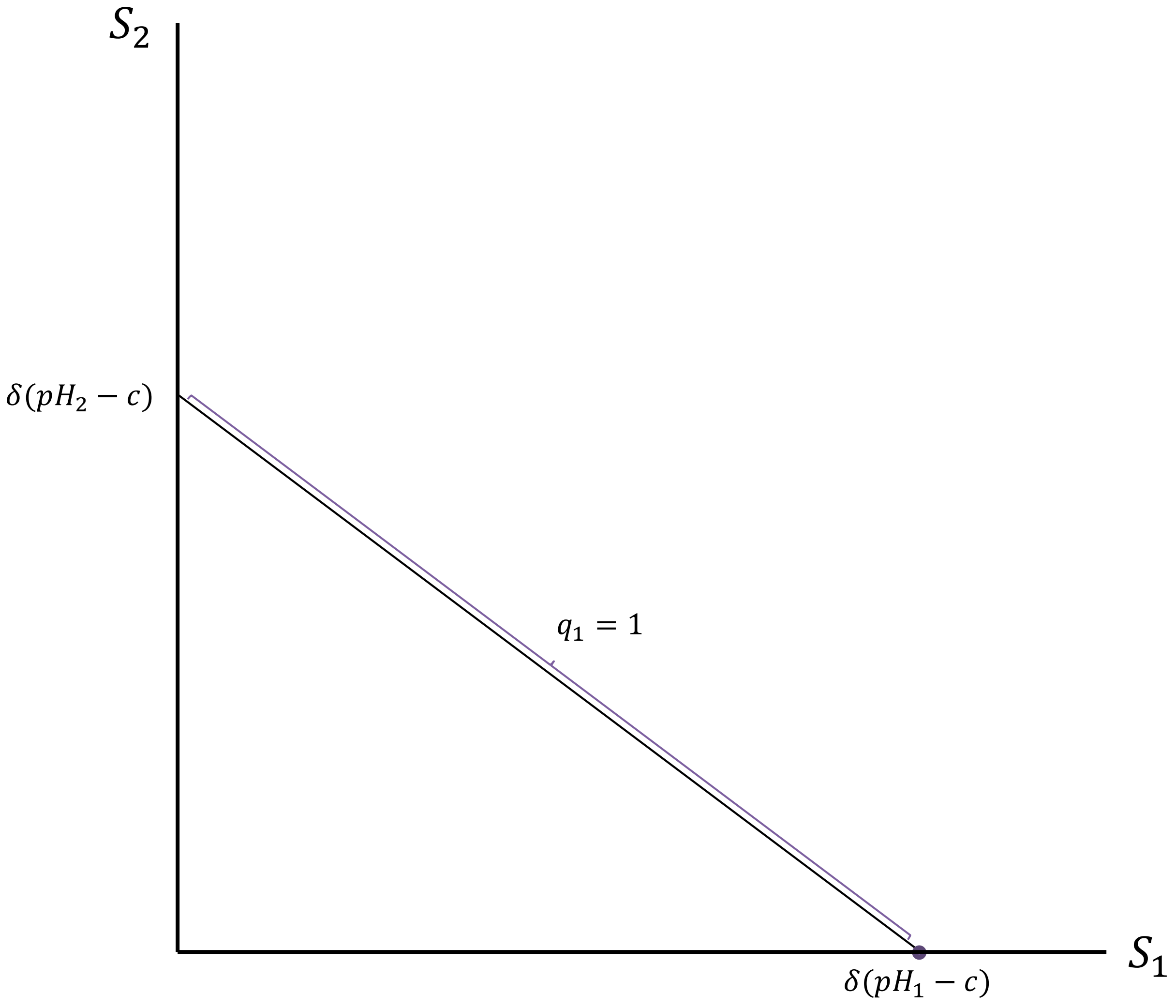


$$0 \leq B_1(y) \leq \delta(pH_1 - c)q_1$$

$$0 \leq B_2(y) \leq \delta(pH_2 - c)q_2$$

OPTION 1: EX POST EFFICIENCY



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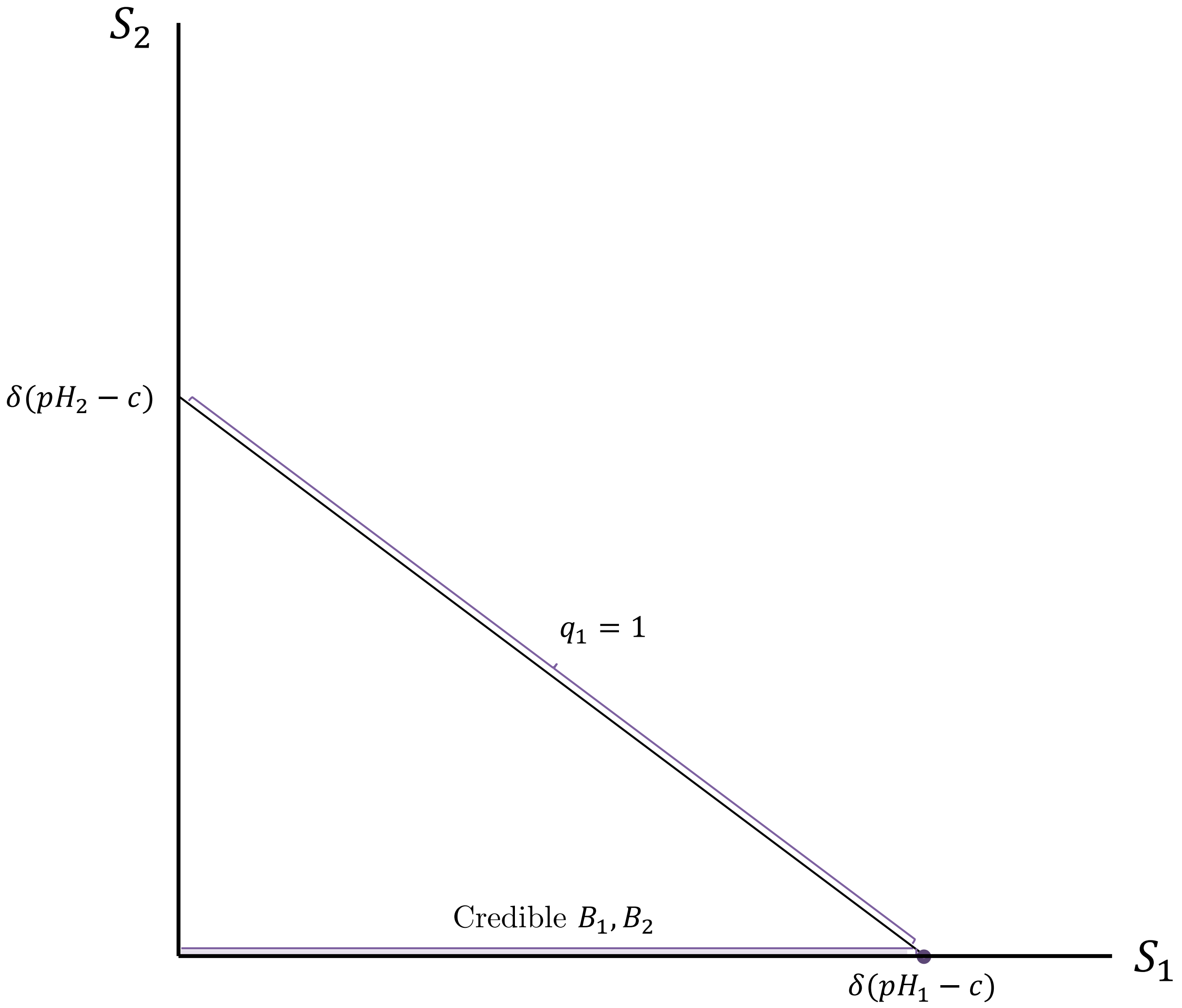

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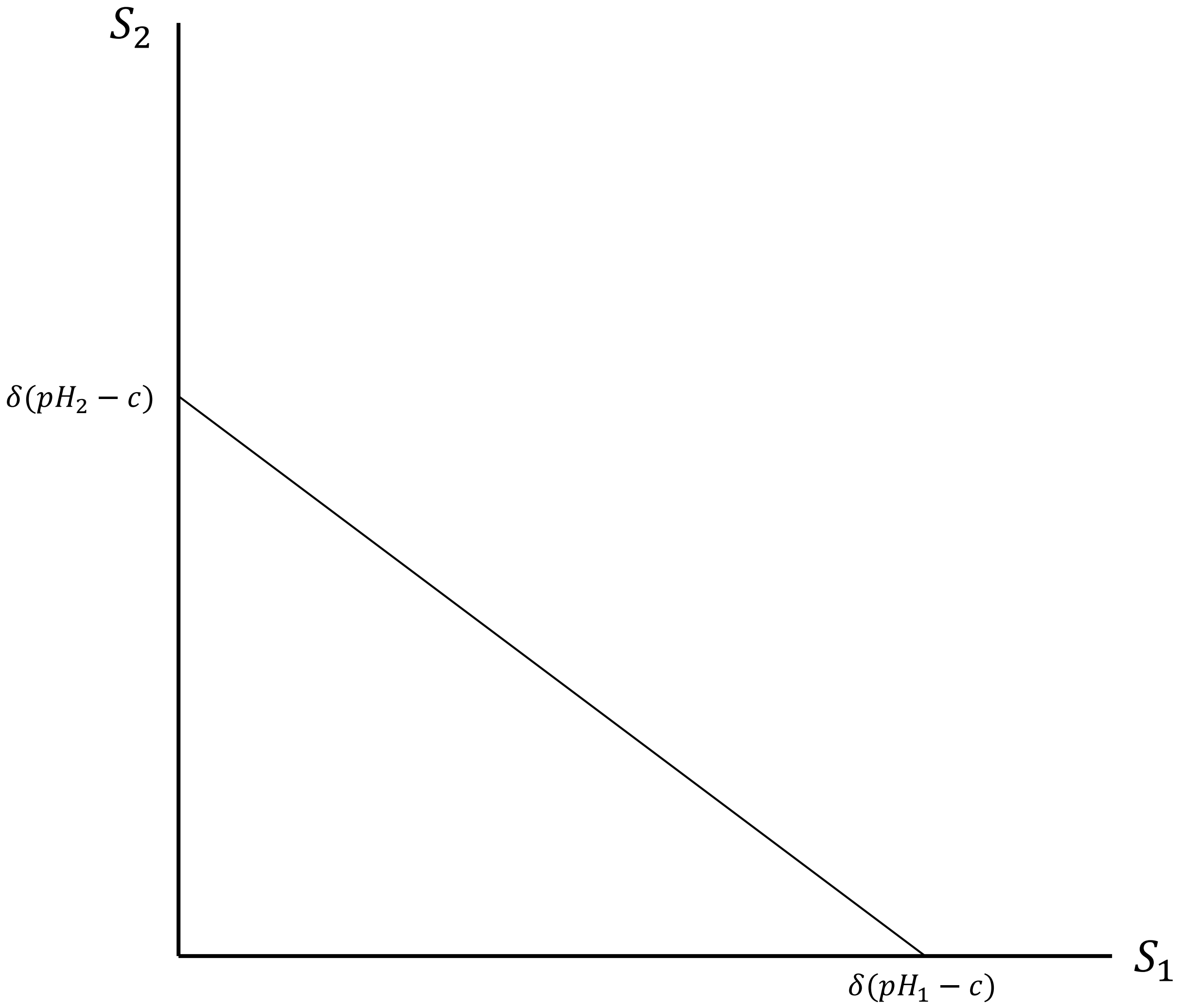
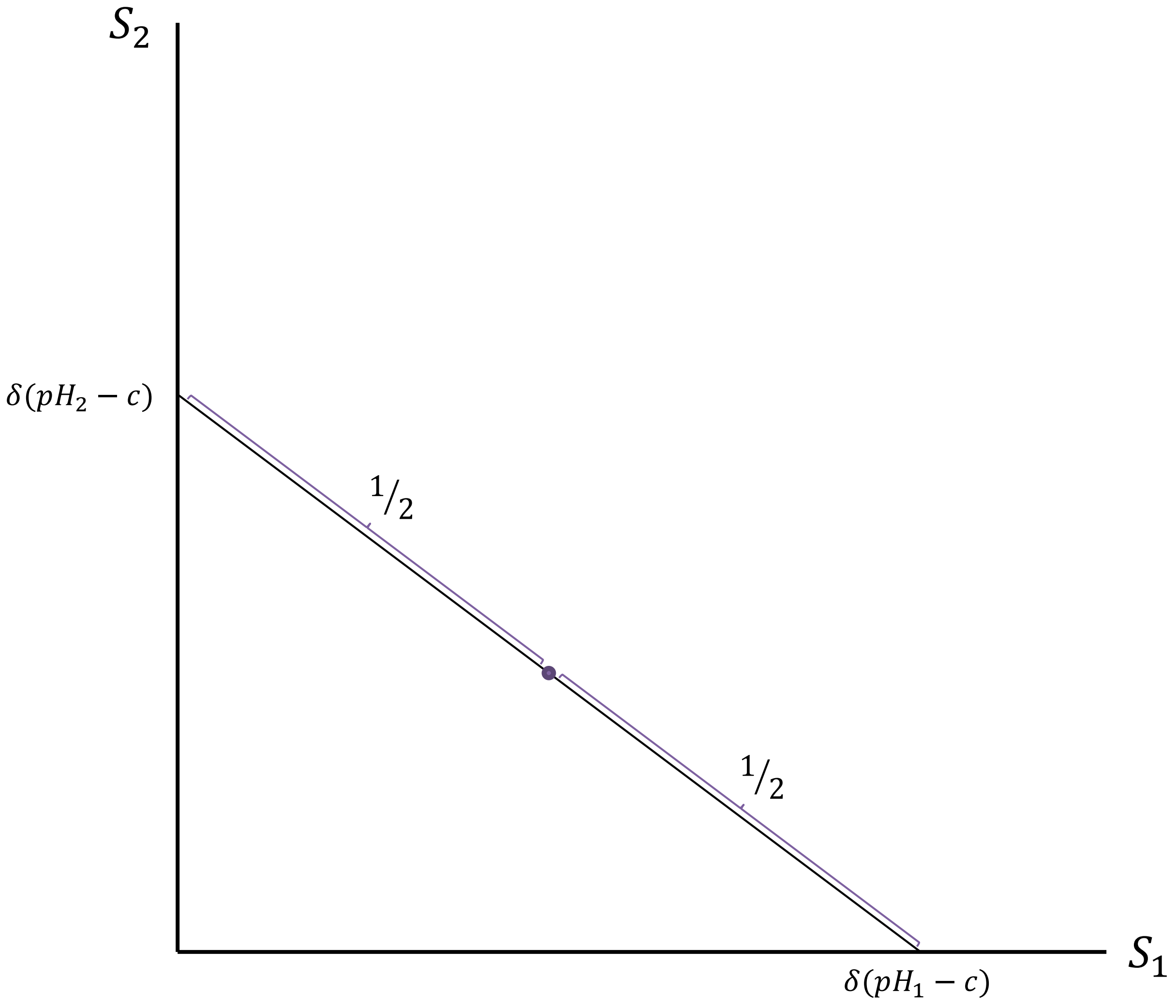
OPTION 2: RANDOMIZATION



Option 2: Randomization



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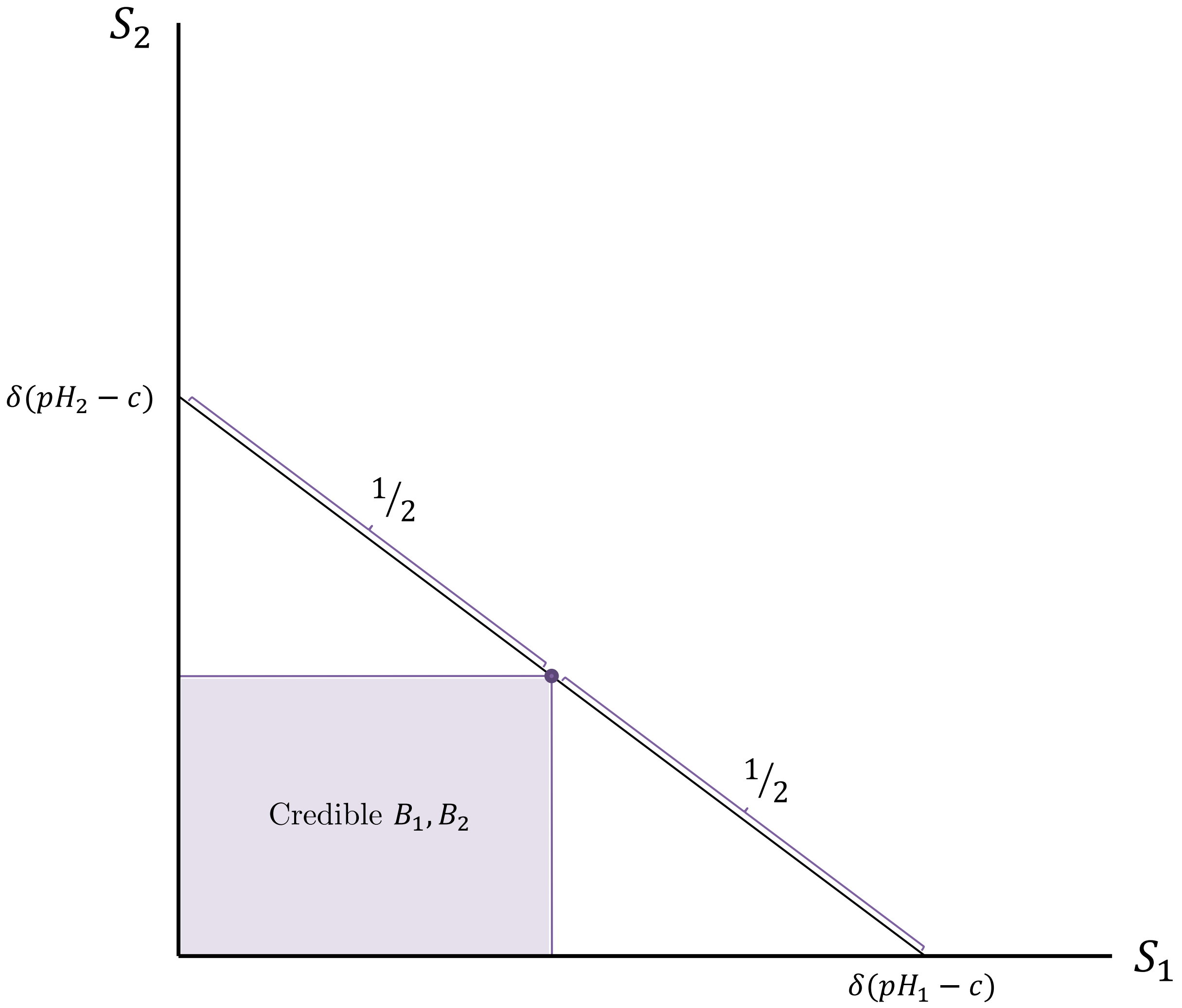

$$0 \leq B_1(y) \leq \delta(pH_1 - c)^1/_2$$

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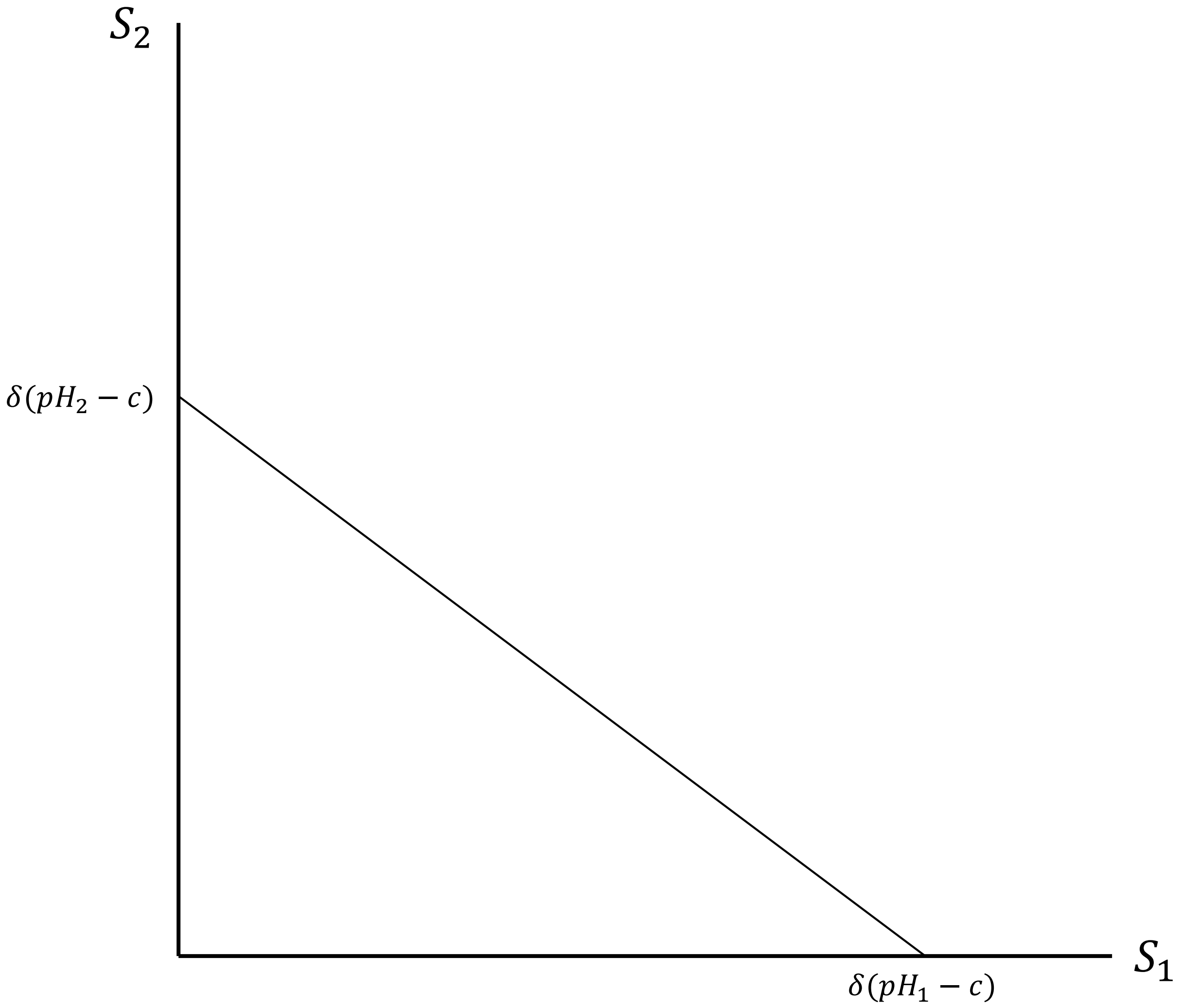
OPTION 3: HISTORY-DEPENDENT INEFFICIENCIES



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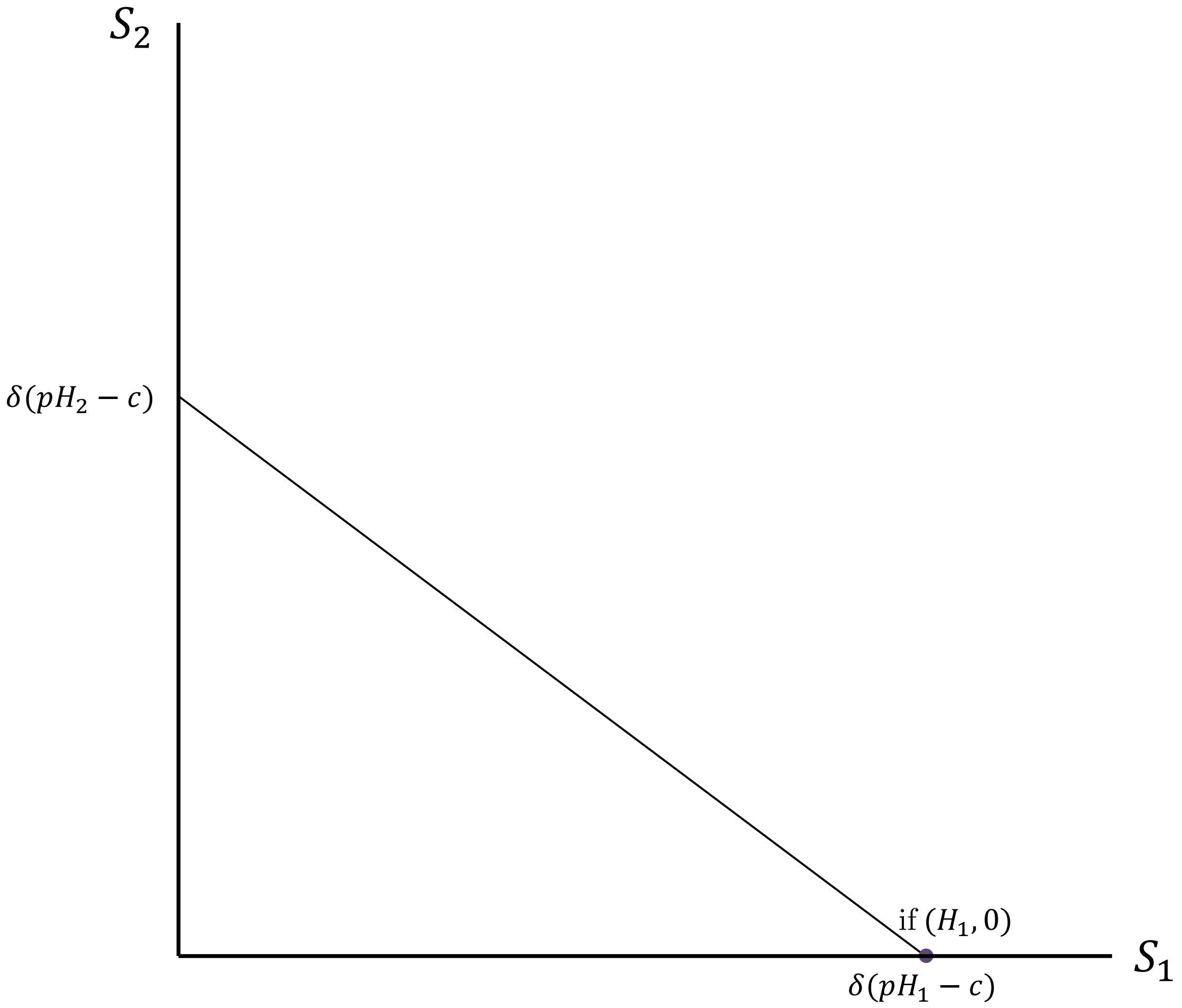

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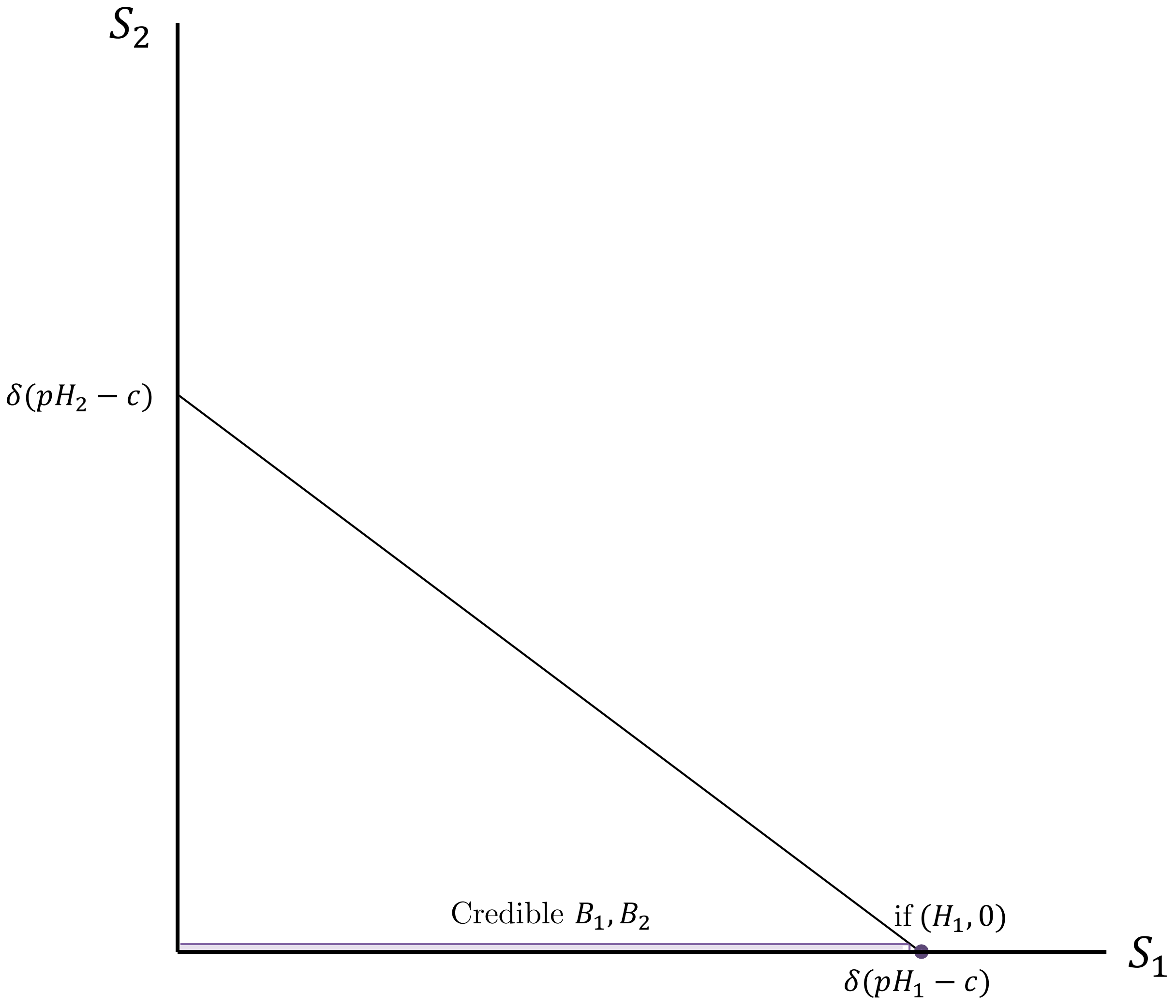

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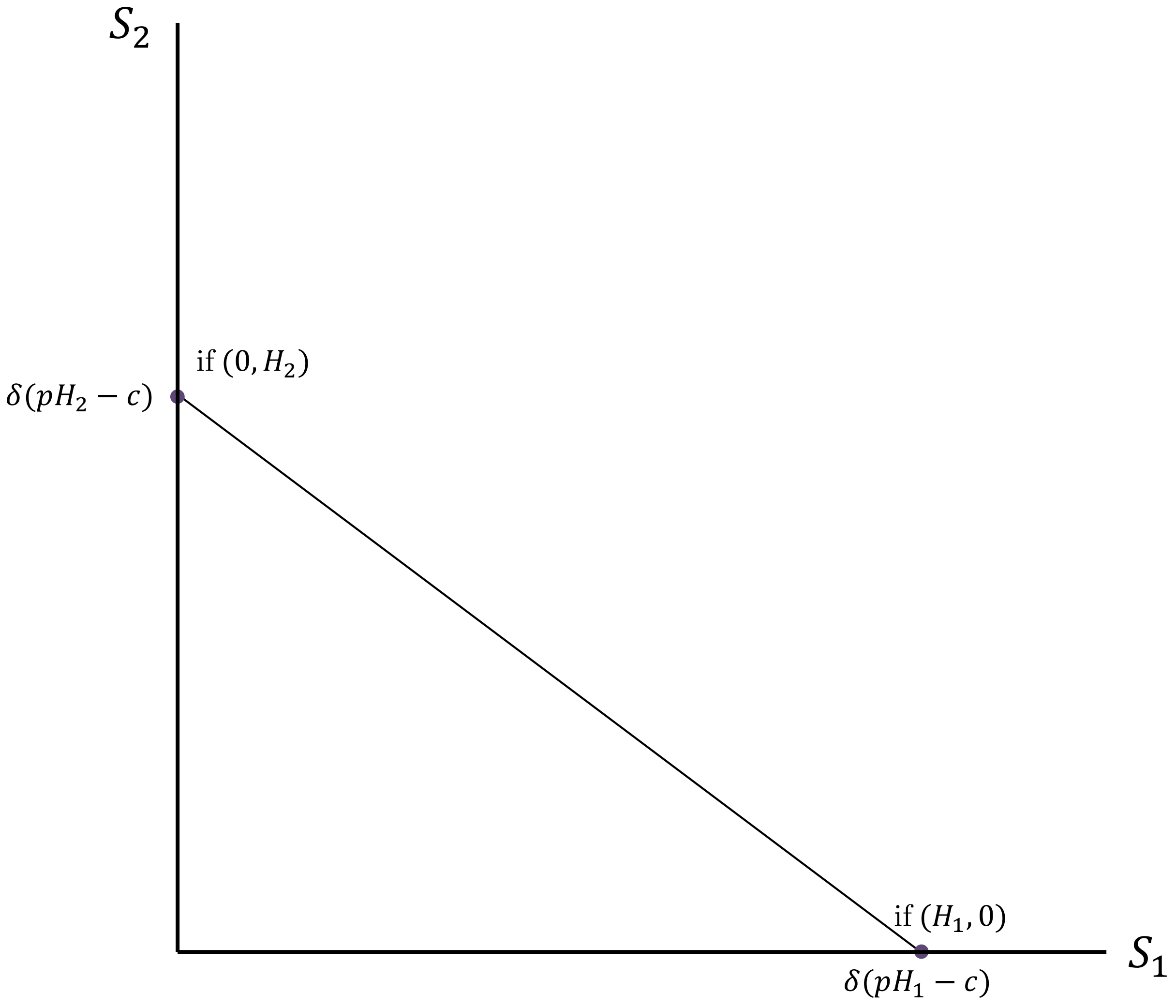

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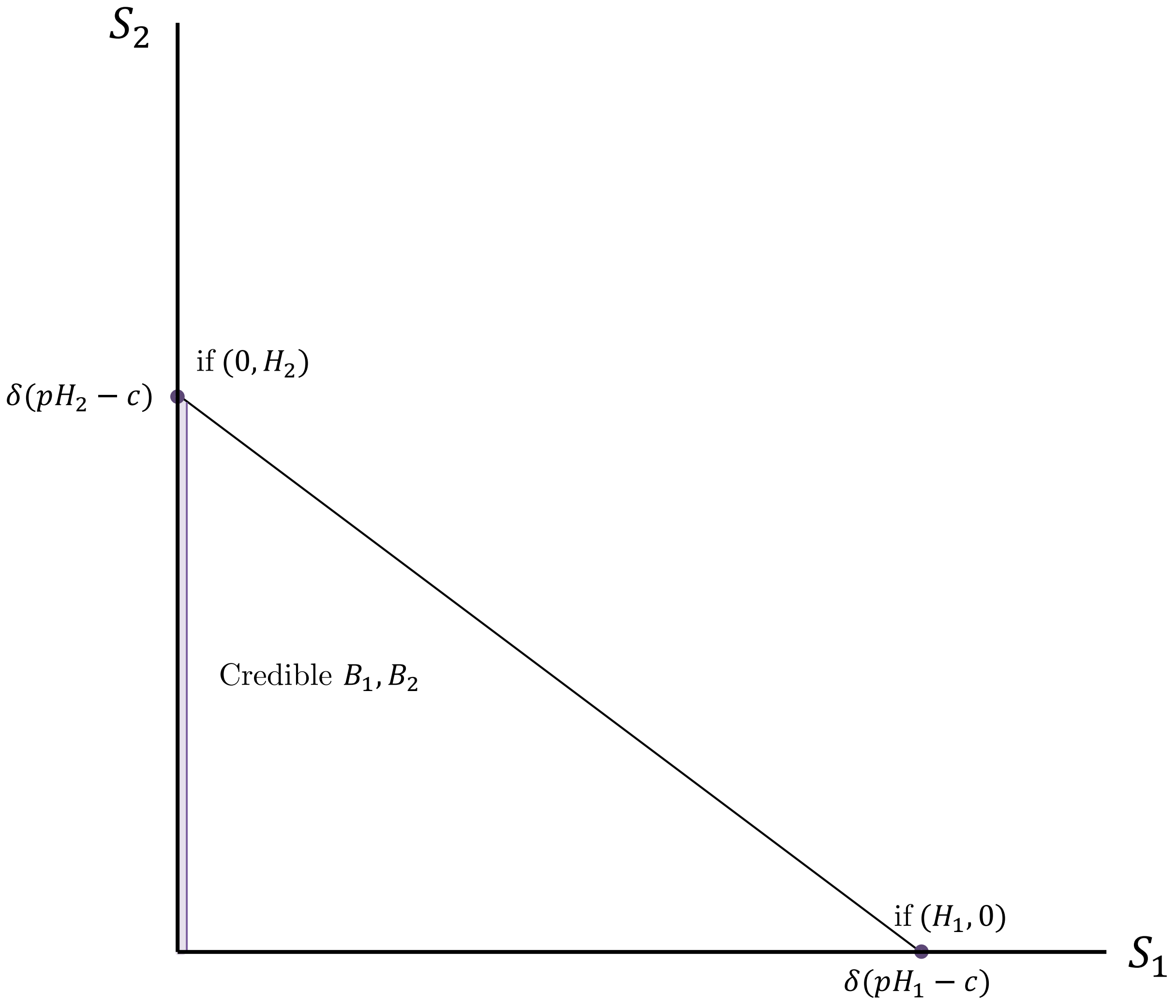

$$0 \leq B_1(y) \leq 0$$

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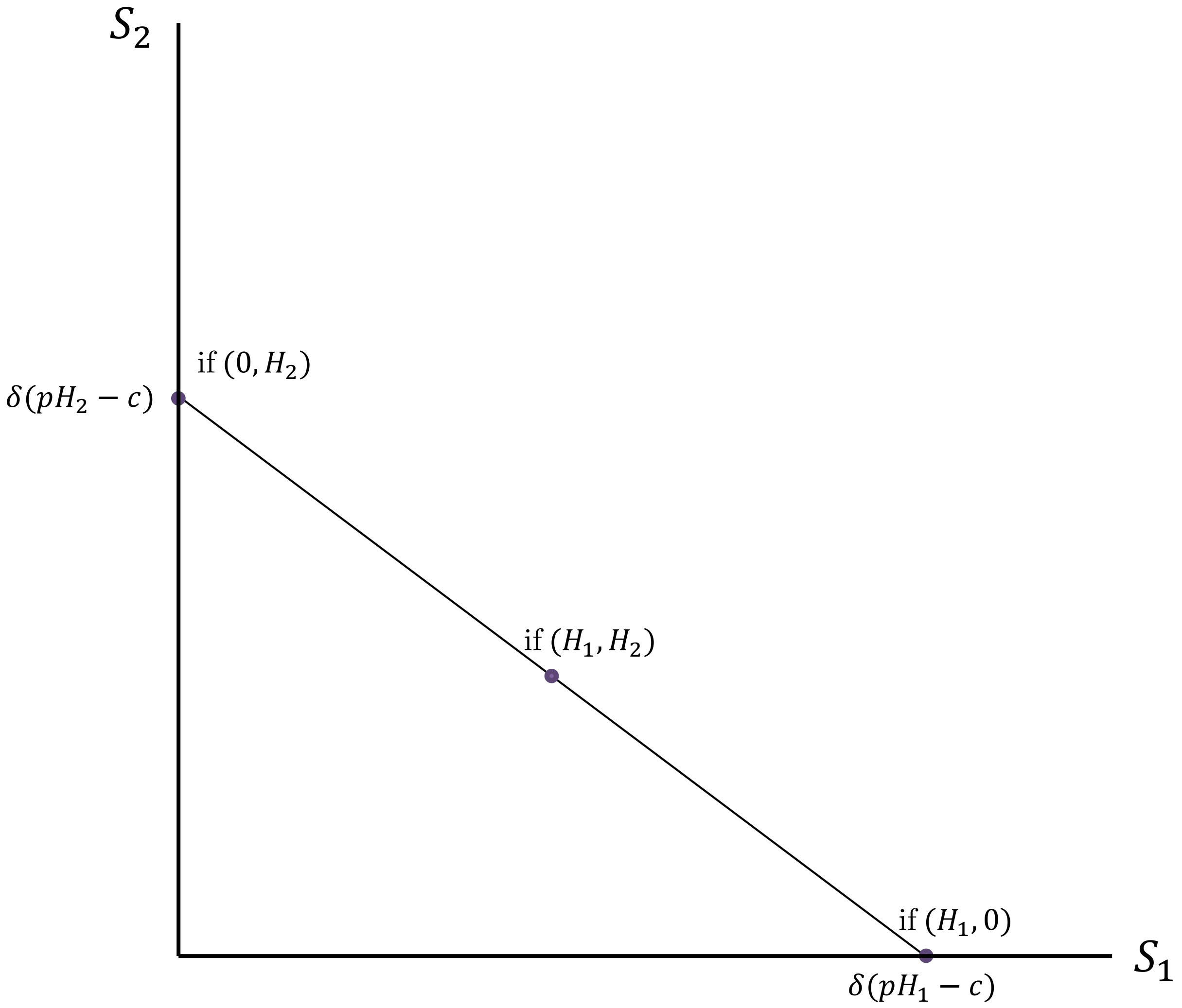

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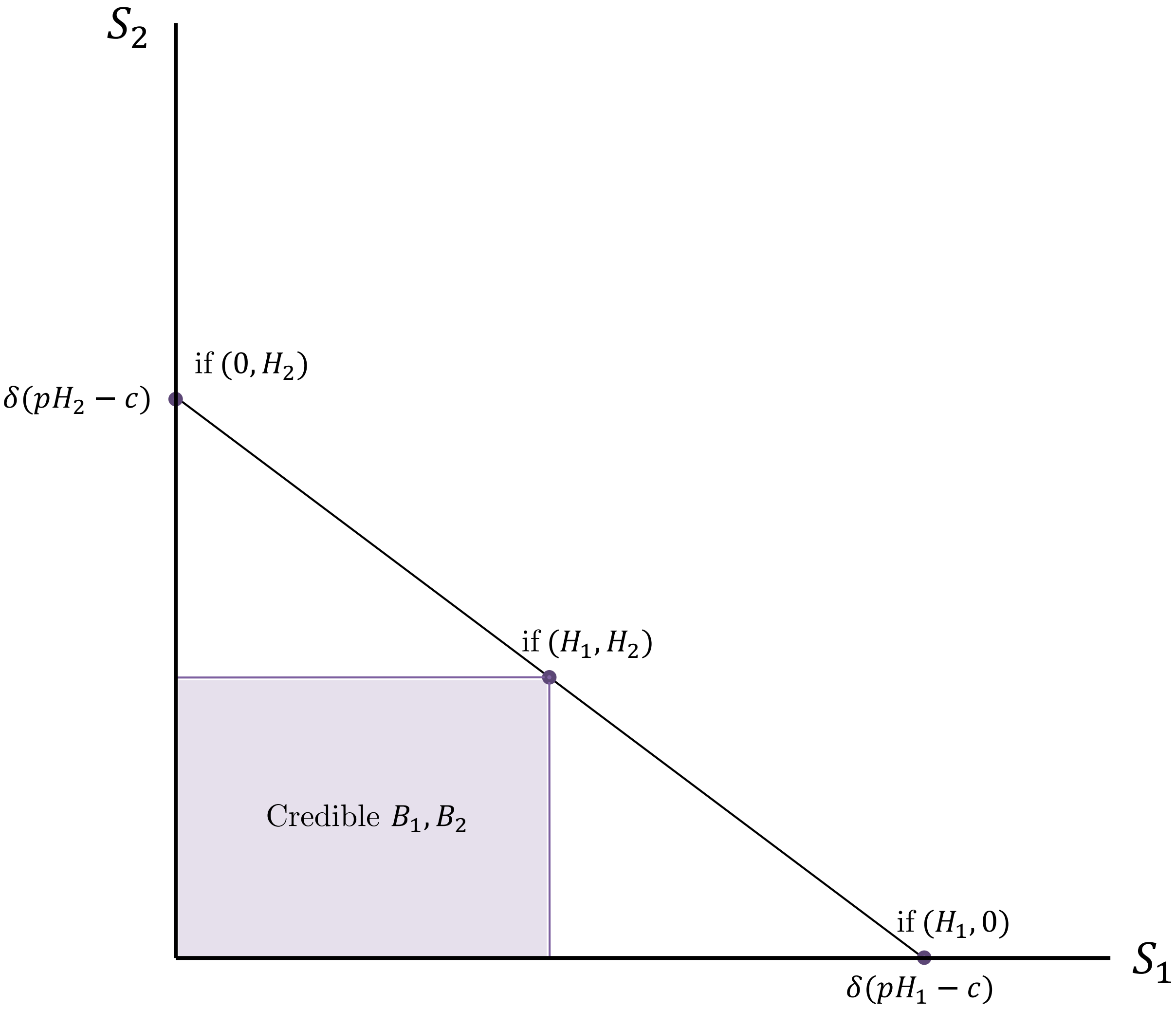

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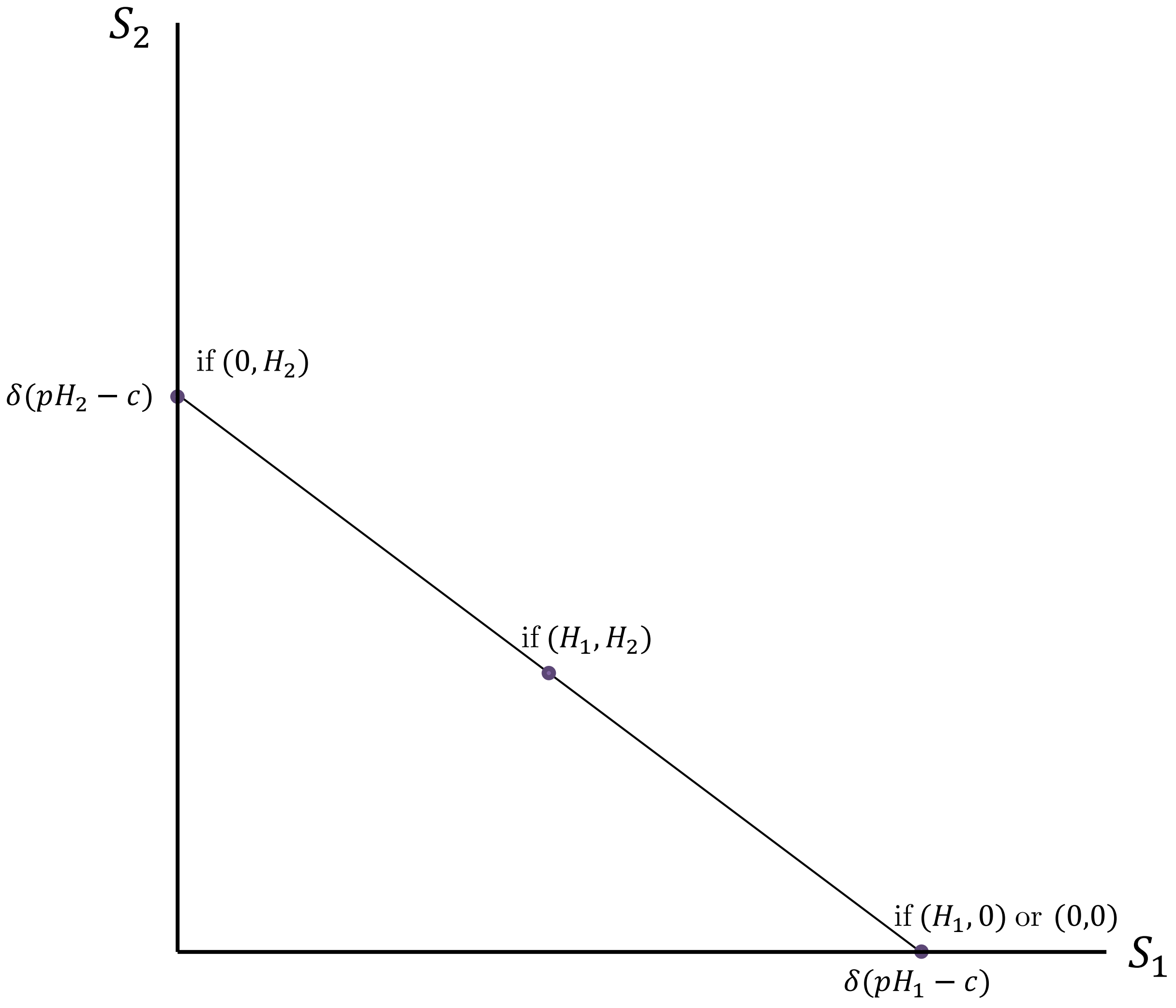

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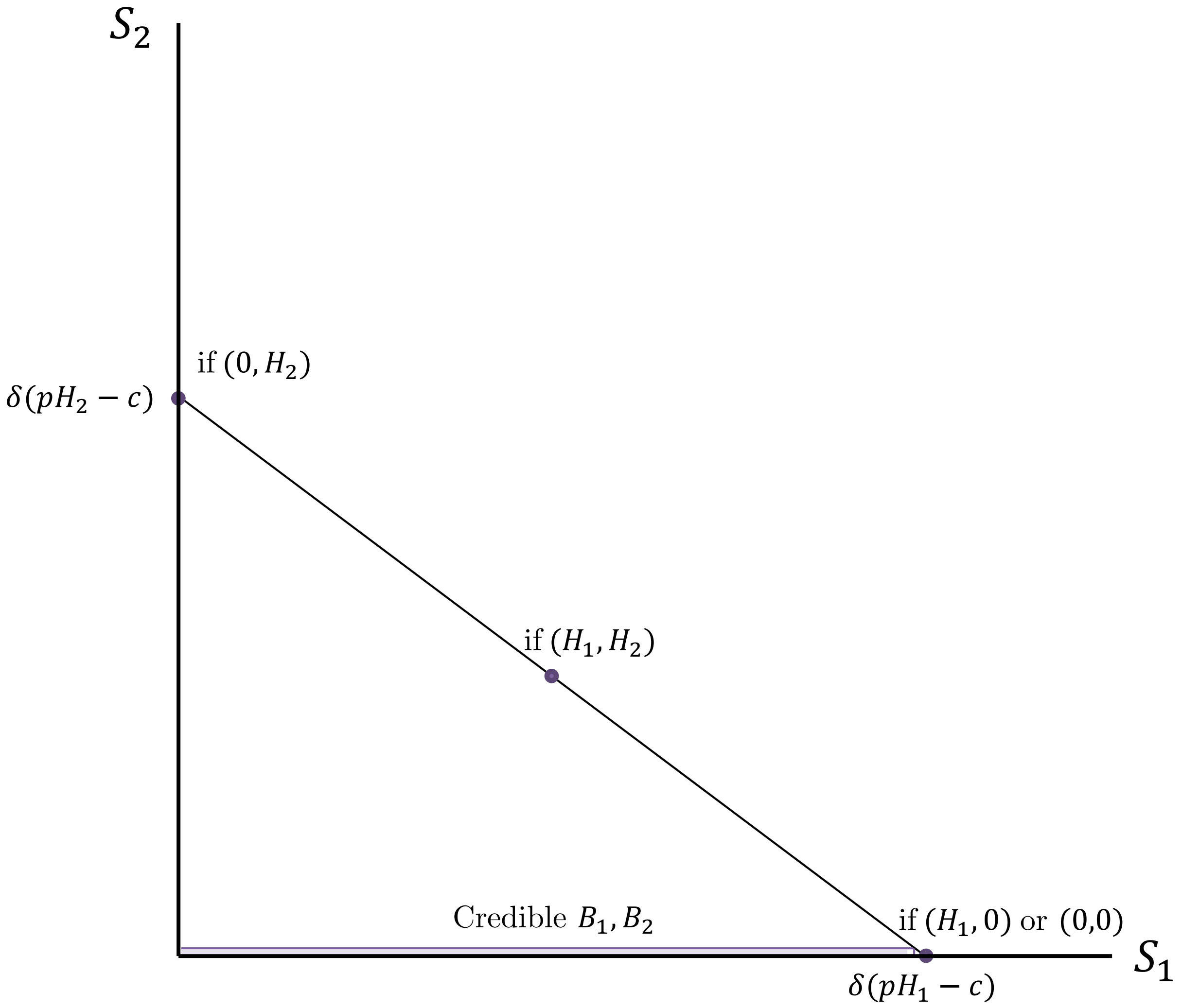

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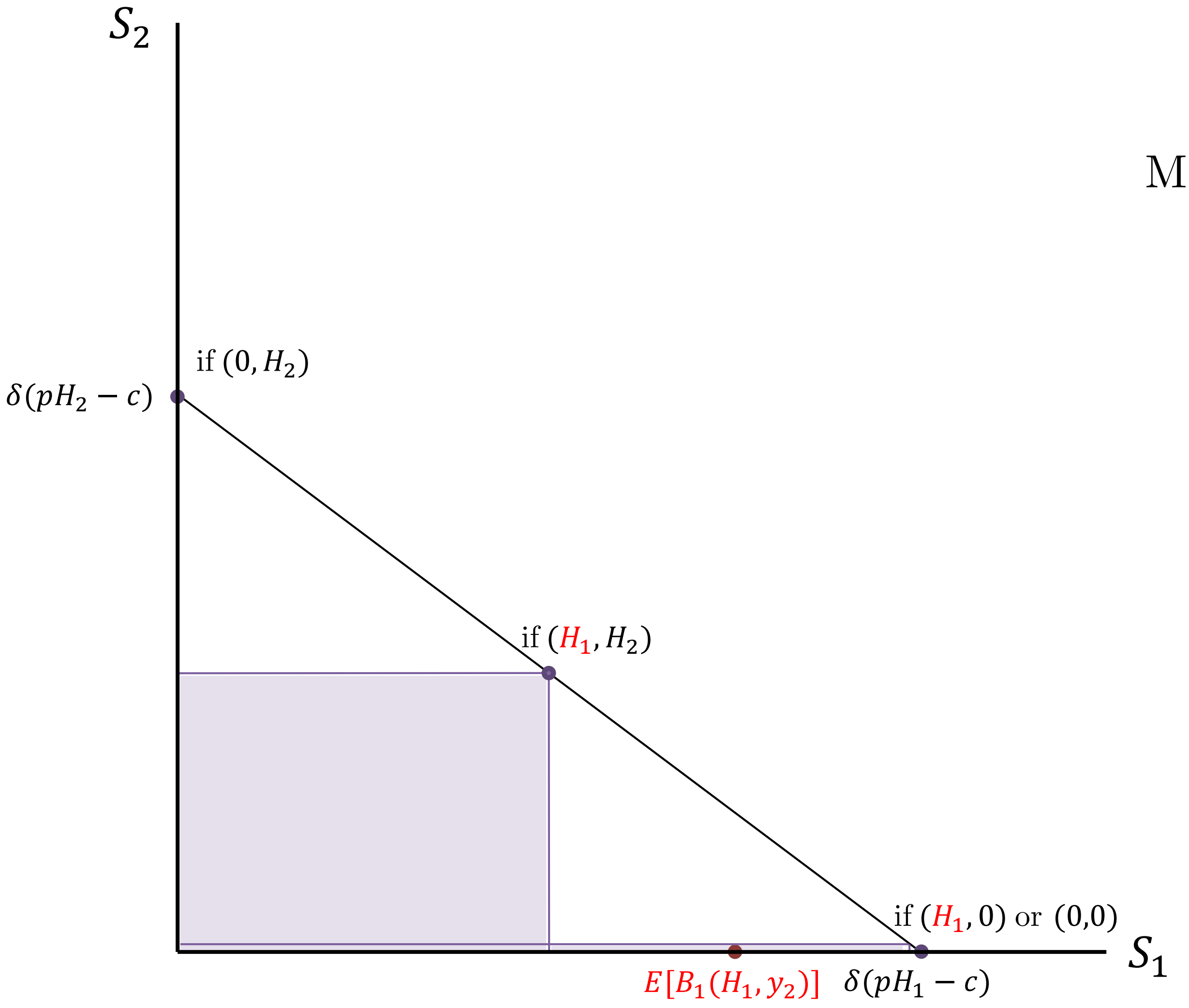

Maximal incentives for Agent 1:

$E[B_1(H_1, y_2)] - 0$

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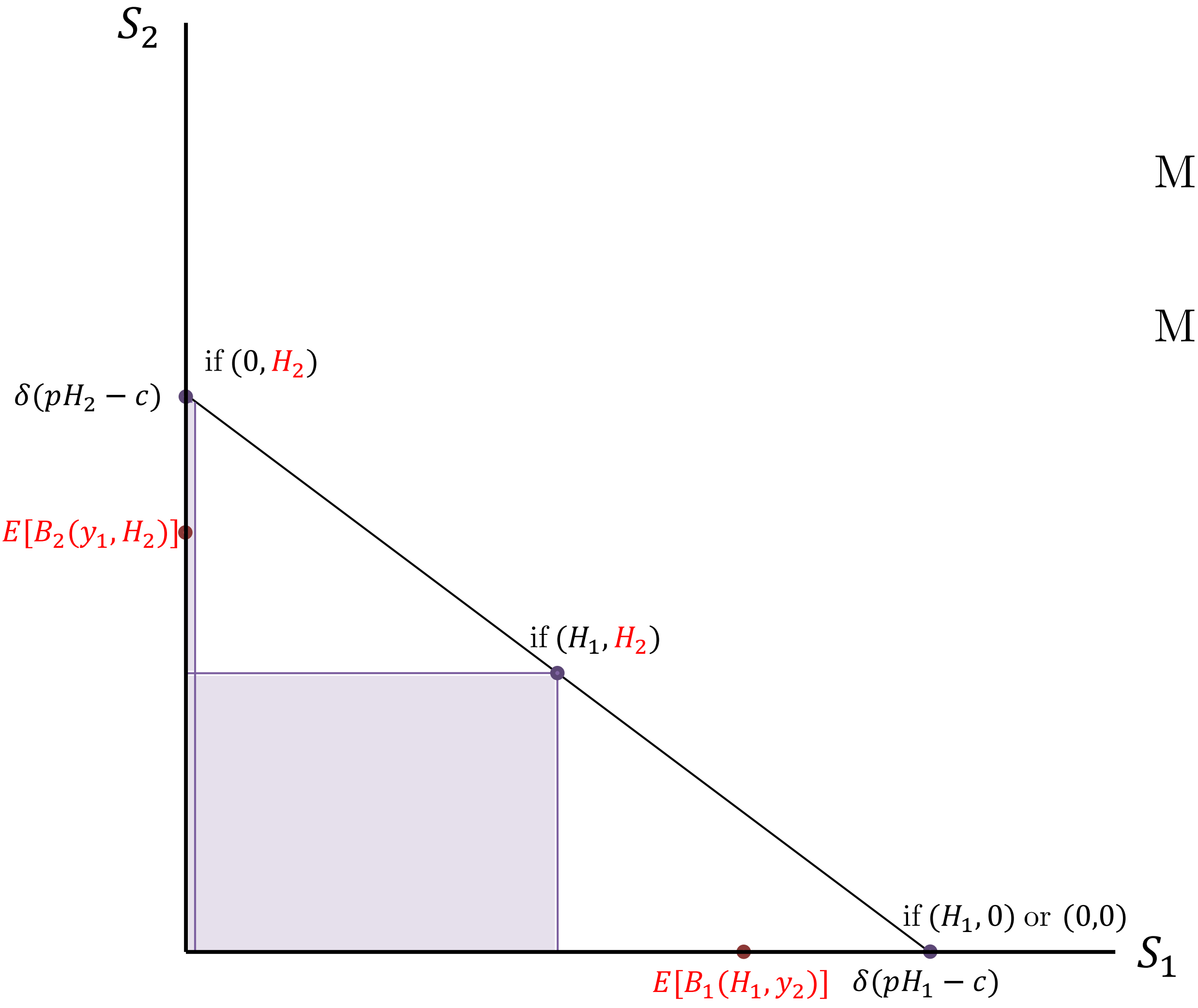


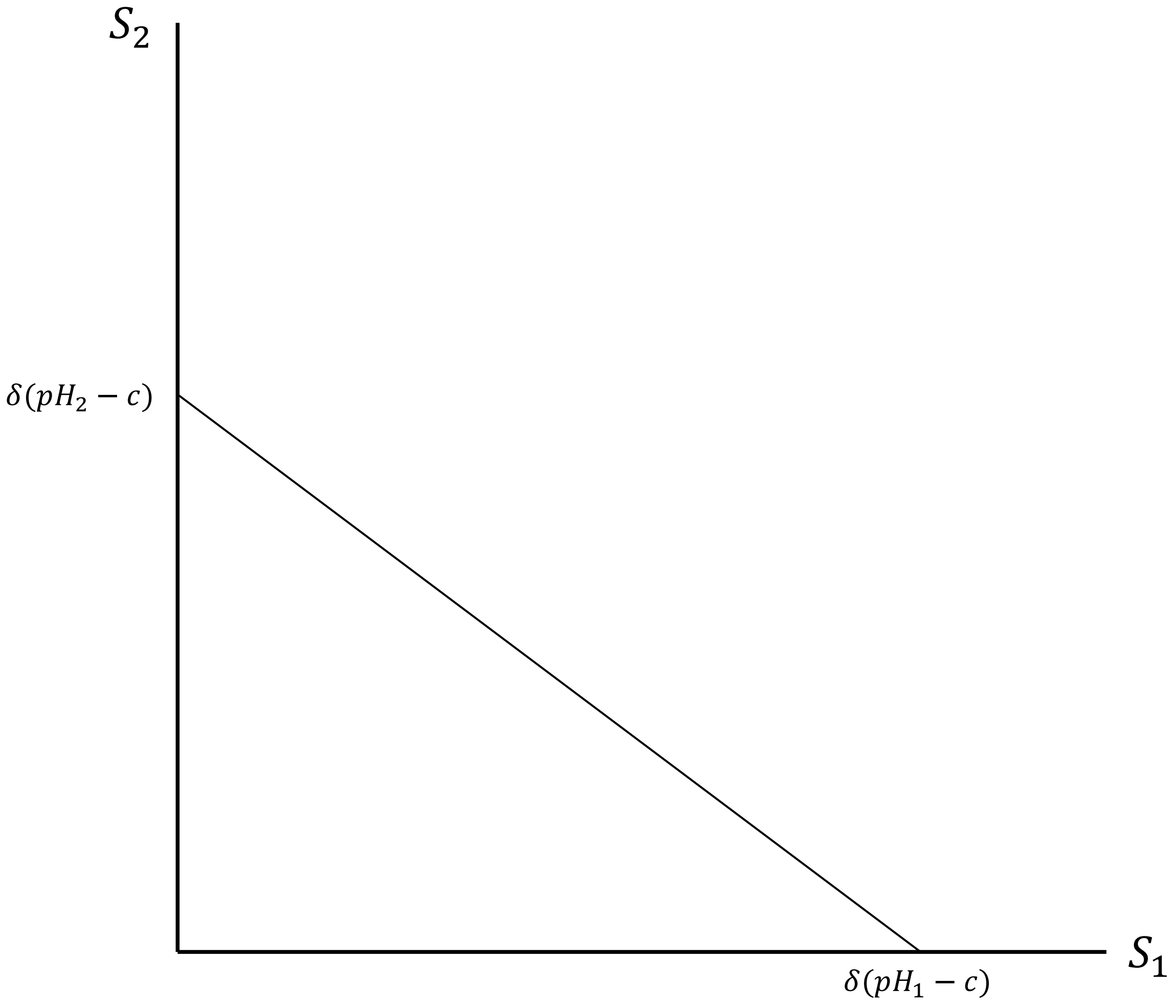
Maximal incentives for Agent 1:

$$E[B_1(H_1, y_2)] - 0$$

Maximal incentives for Agent 2:

$$E[B_2(y_1, H_2)] - 0$$

WHAT IF EVERYTHING (BUT e) IS PUBLIC?



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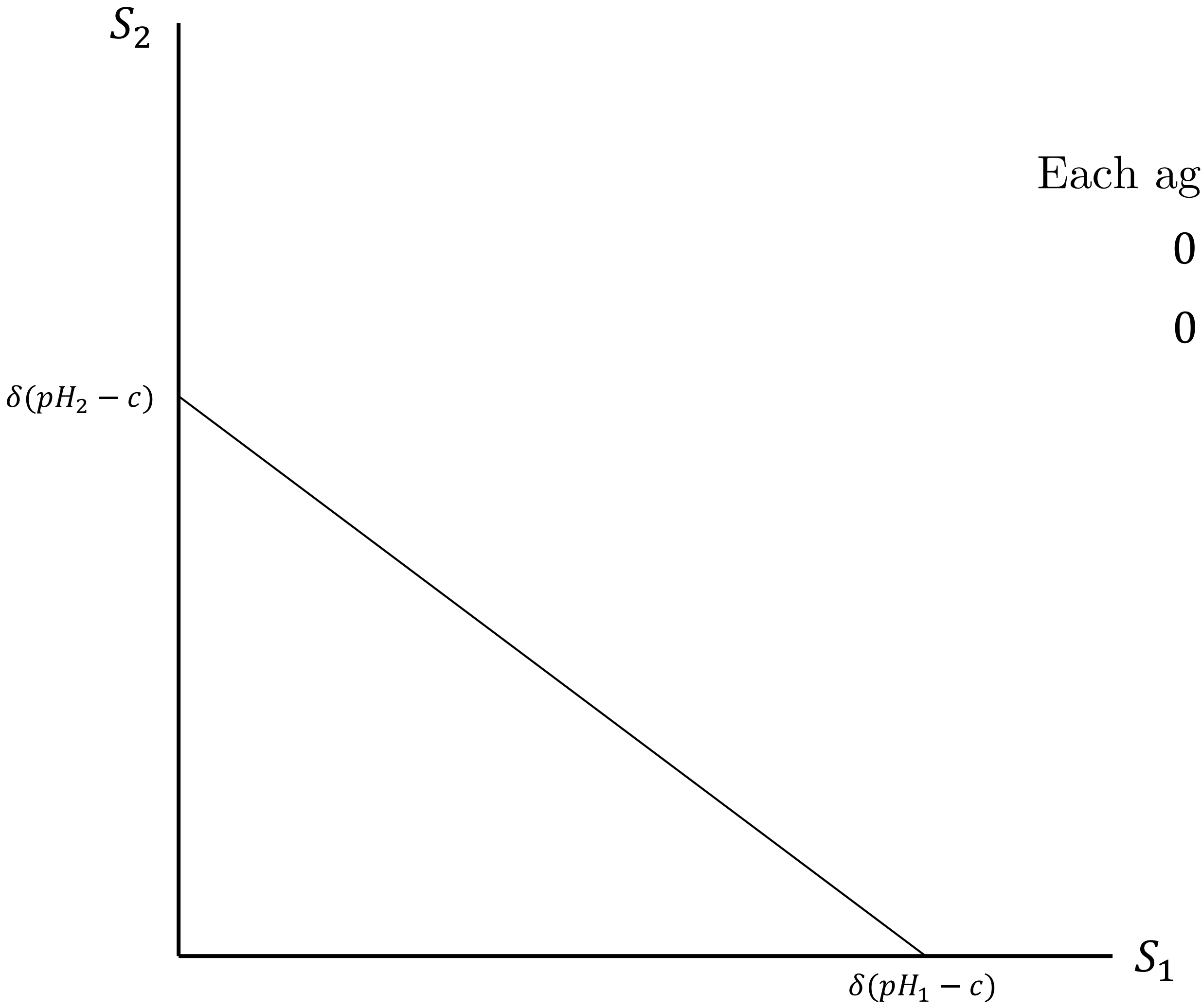

Each agent can walk away:

$$0 \leq B_1$$

$$0 \leq B_2$$

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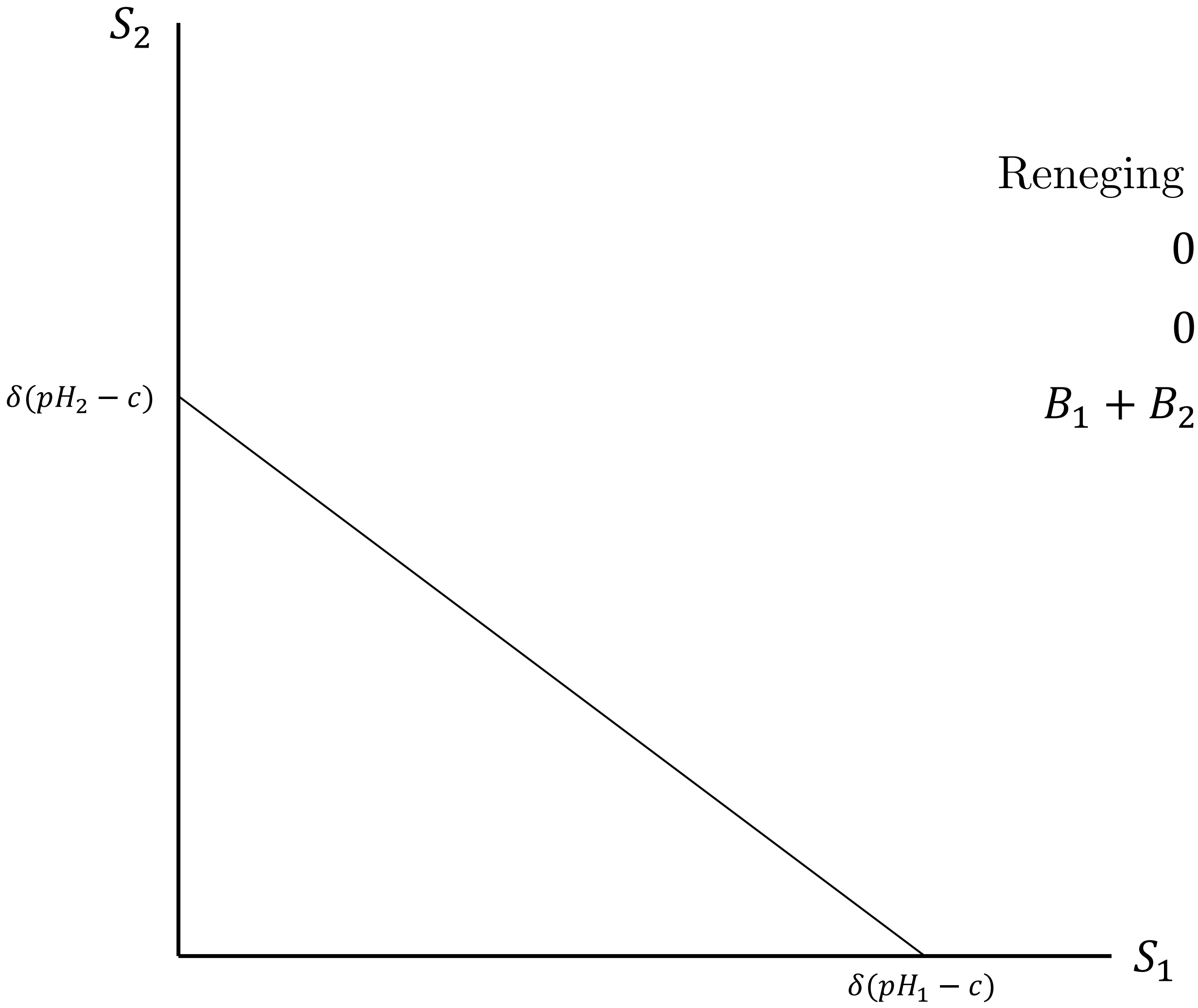

Reneging can be jointly punished:

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$$B_1 + B_2 \leq \delta(p(q_1 H_1 + q_2 H_2) - c)$$

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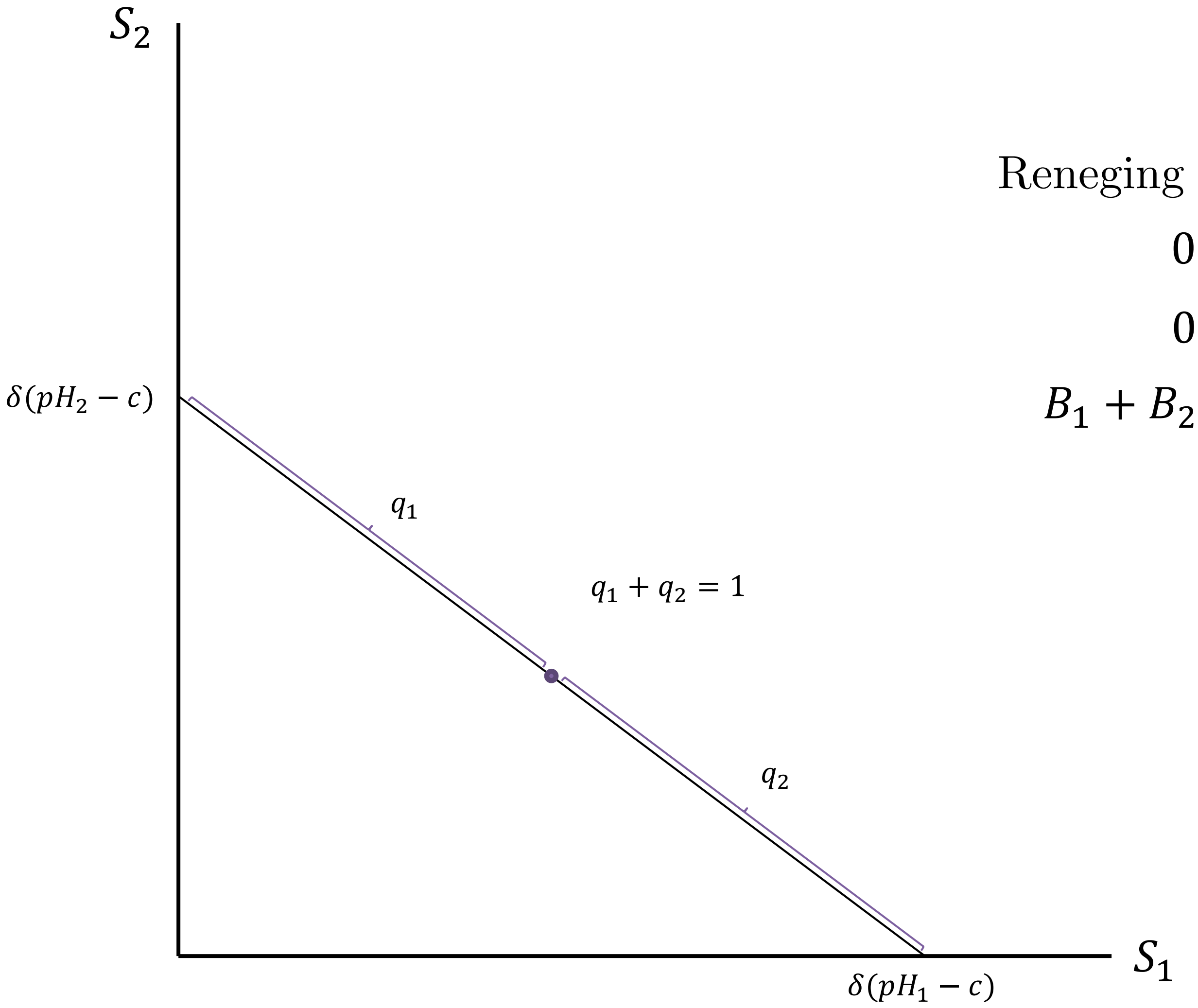

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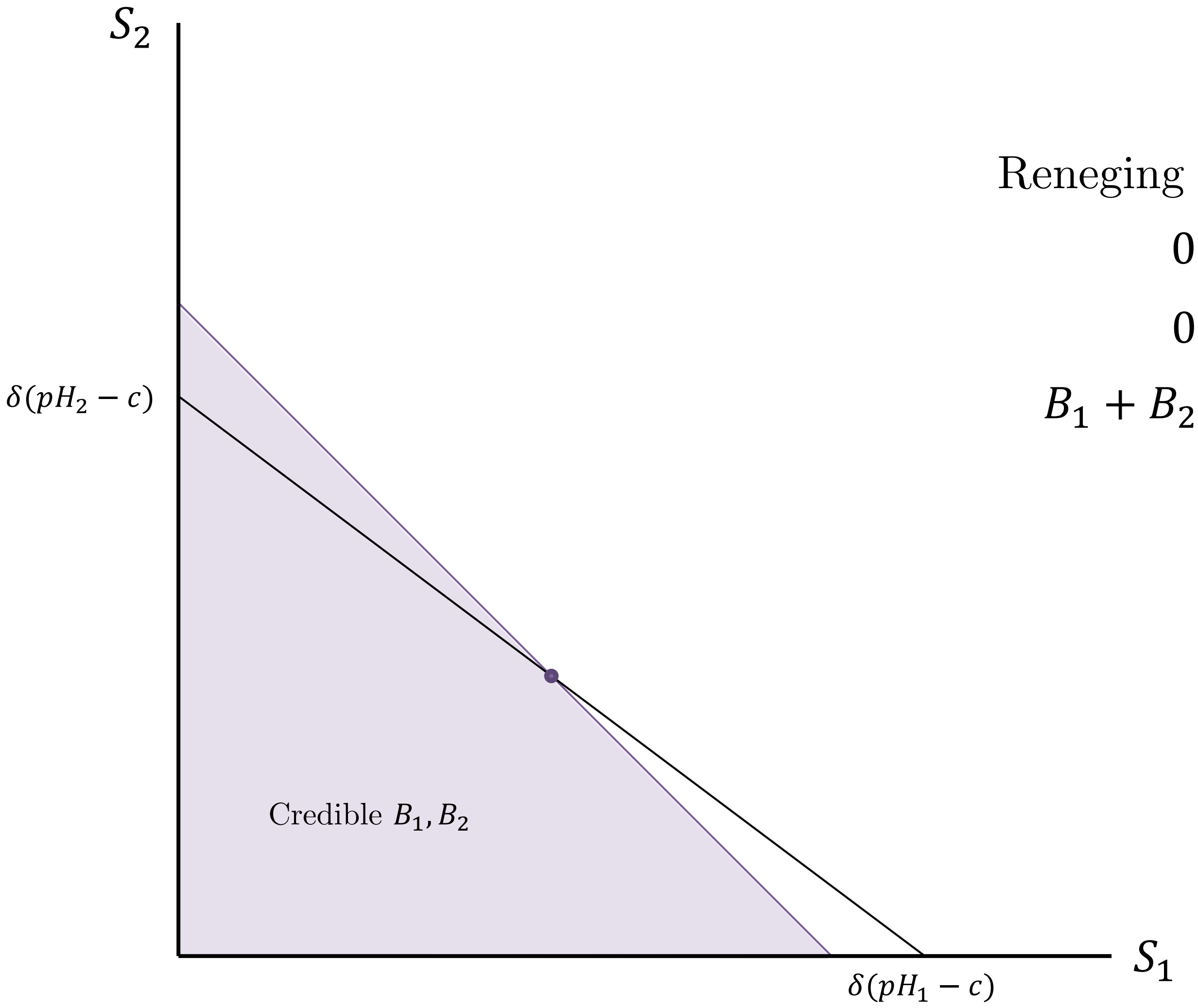

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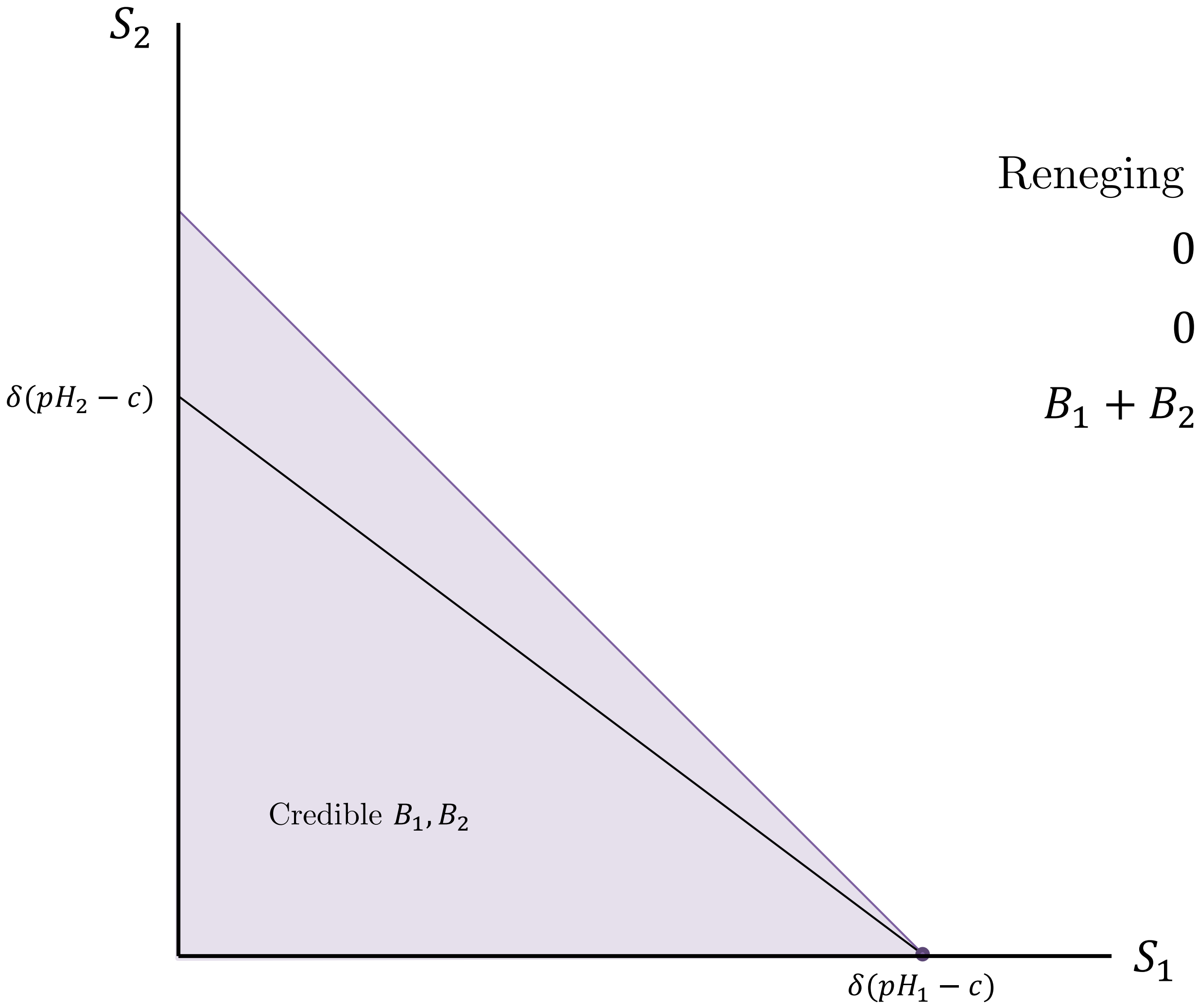
NO BIASES IF MONITORING IS PUBLIC



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MODEL INGREDIENTS

One principal, N agents: risk-neutral, common discount factor δ

Public state of the world θ

Principal makes **decision** d from set D

Agent i's **output:** $P_i(y_i | \underset{\substack{\nearrow \\ \text{effort}}}{e_i}, \underset{\substack{\uparrow \\ \text{state}}}{\theta}, \underset{\substack{\nwarrow \\ \text{decision}}}{d})$

A **policy** is a history-contingent decision plan

EXAMPLES OF DECISIONS

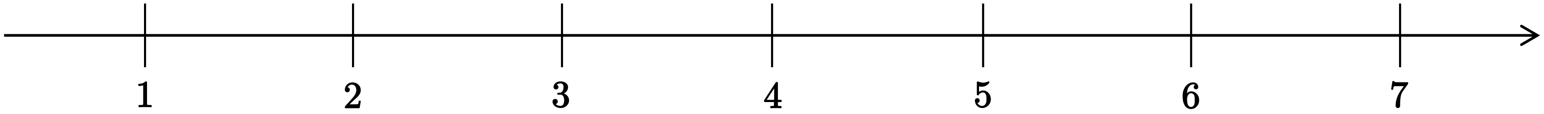
Hiring / Firing: D = agents available; $\boldsymbol{\theta}$ = demand; $\boldsymbol{d}$ = agents hired

Promotion: D = set of agents up for promotion; $\boldsymbol{d}$ = agent promoted

Irreversible investment: D = set of agents (if no investment yet), chosen agent otherwise; $\boldsymbol{d}$ = agent chosen for investment

Sourcing decision: D = set of available suppliers; $\boldsymbol{\theta}$ = each supplier's productivity; $\boldsymbol{d}$ = supplier chosen

STAGE GAME



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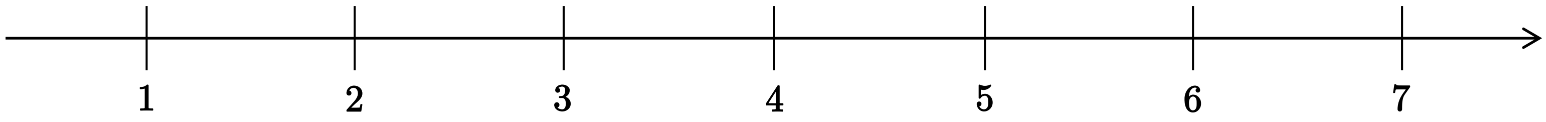
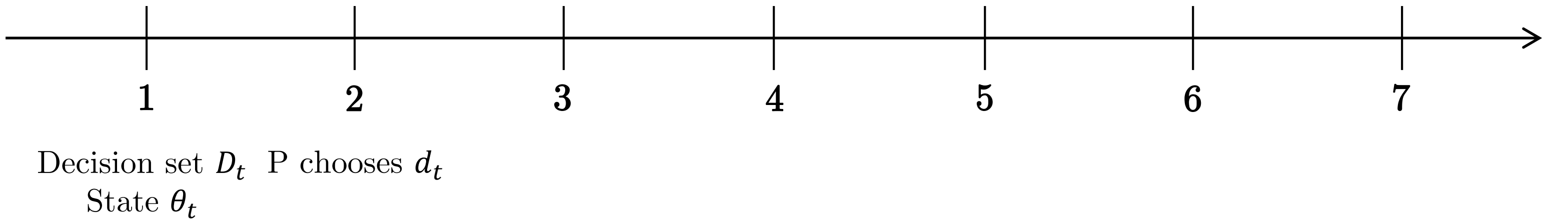


Decision set D_t
State θ_t

1: Decision set D_t and state θ_t drawn from $F(\cdot|\{\theta_{t'}, D_{t'}, d_{t'}\}_{t'=0}^{t-1})$.
Publicly observed.

STAGE GAME



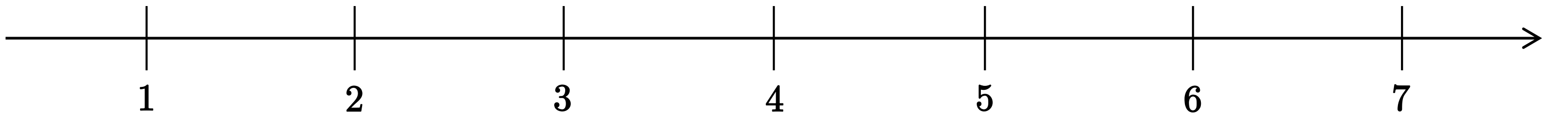
Decision set D_t

State θ_t

P chooses d_t

2: Principal chooses decision $d_t \in D_t$. Publicly observed.

STAGE GAME



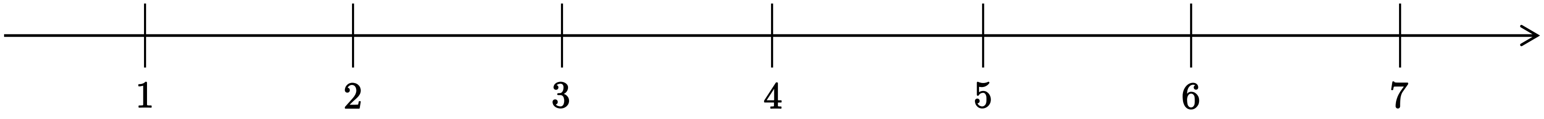
Decision set D_t
State θ_t

P chooses d_t

Transfers $w_{i,t}$
Messages $m_{i,t}$

3: Principal and each agent pay each other $w_{i,t} \in \mathbb{R}$. Principal sends messages $\{m_{i,t}\}_{i=1}^{N}$ to each agent. Bilaterally observed.

STAGE GAME



Decision set D_t
State θ_t

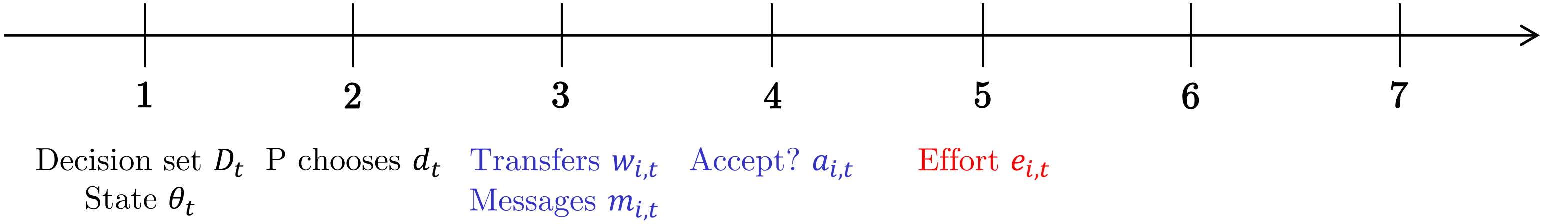
P chooses d_t

Transfers $w_{i,t}$
Messages $m_{i,t}$

Accept? $a_{i,t}$

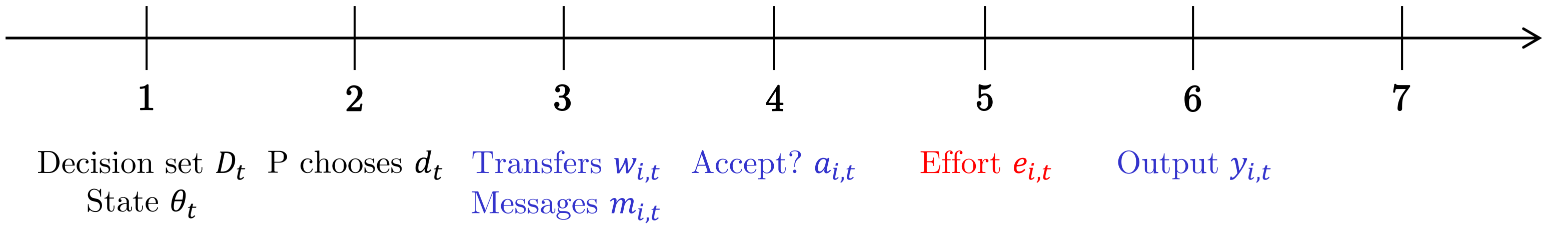
4: Each agent i accepts or rejects, $a_{i,t} \in \{0,1\}$. Outside option $\bar{u}_i(d_t, \theta_t) \geq 0$ results in $y_{i,t} = 0$. Bilaterally observed.

STAGE GAME



5: If i accepts, chooses effort $e_{i,t} \geq 0$ at cost $c(\cdot)$. Privately observed.

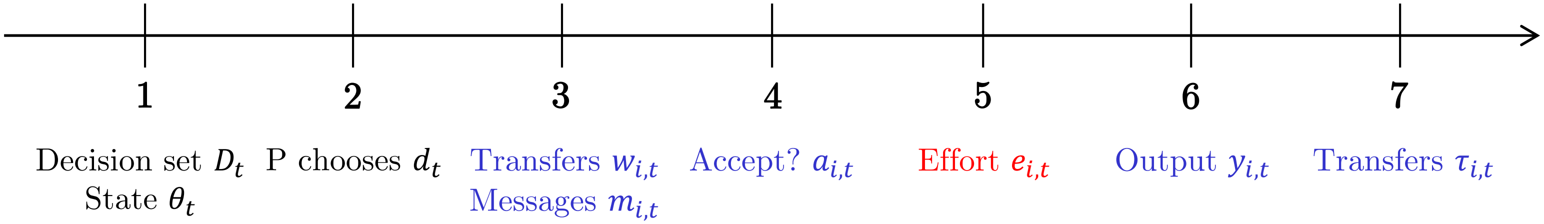
Stage Game



6: Output $y_{i,t} \in \mathbb{R}_+$ realized according to $P_i(\cdot \,|e_{i,t}, \theta_t, d_t)$.

Bilaterally observed.

STAGE GAME



1: Decision set D_t, State θ_t

2: P chooses d_t

3: Transfers $w_{i,t}$, Messages $m_{i,t}$

4: Accept? $a_{i,t}$

5: Effort $e_{i,t}$

6: Output $y_{i,t}$

7: Transfers $\tau_{i,t}$

7: Principal and agent i exchange (net) transfers $\tau_{i,t} \in \mathbb{R}$.

Bilaterally observed.

PAYOFFS AND INFORMATION

$$\pi_t = (1-\delta)\sum_{i\leq N}(y_{i,t} - w_{i,t} - \tau_{i,t})$$

$$u_{i,t} = (1-\delta)(w_{i,t} + \tau_{i,t} - (a_{i,t}c(e_{i,t}) - (1-a_{i,t})\bar{u}_i))$$

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Dyad-surplus: $S_{i,t} = \sum_{t'\geq t} \delta^{t'-t}(1-\delta)(y_{i,t'} - C_{i,t'})$

Histories: h_0^t at start of period, h_x^t after variable x, agent i sees $\phi_i(h_x^t)$

RECURSIVE EQUILIBRIUM

A Perfect Bayesian Equilibrium σ^* is a **recursive equilibrium** if, for each h_0^t on equilibrium path, $\sigma^*|h_0^t$ is a Perfect Bayesian Equilibrium.

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Implications for behavior:

- Agent i's effort IC constraint conditions on h_0^t , not $\phi_i(h_0^t)$
- When paying $\tau_{i,t}$, agent i has Bayesian expectations over $y_{-i,t}$

SURPLUS-MAXIMIZING RELATIONAL CONTRACTS

A recursive equilibrium σ^* is **surplus-maximizing** if it maximizes ex ante total surplus among recursive equilibria. It is **sequentially surplus-maximizing** if $\sigma^*|h_0^t$ is surplus-maximizing for every on-path history h_0^t.

A **biased** decision is not sequentially surplus-maximizing

A policy is **backward-looking** if it involves on-path biased decisions

AGENDA

- Illustrative Example
- The General Model
- Main Results
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MAIN RESULTS

Necessary and sufficient conditions for relational contract to be self-enforcing: IC and agent-specific dynamic enforcement constraints

Biased decisions are surplus-maximizing in smooth games

CREDIBLE REWARD SCHEMES

Given a rel. con. σ, a reward scheme B is **credible in $\boldsymbol{\sigma}$** if it satisfies:

1. Incentive compatibility: for each i

$$(a_{i,t}, e_{i,t}) \in \underset{a,e}{\operatorname{argmax}}\, E_y[B_i(y)|a,e] - (1-\delta)C_i$$

2. Dynamic enforcement: for each i and for each on-path h_y^t

$$\delta E[\bar{U}_i] \leq B_i(y_t) \leq \delta E[S_{i,t+1}|y_{i,t}]$$

NECESSARY AND SUFFICIENT CONDITIONS

1. If σ^* is a self-enforcing relational contract, then there exists a reward scheme B^* that is credible in σ^*.

2. If σ is a relational contract with a credible reward scheme B, then there is a self-enforcing relational contract σ^* inducing same joint distribution over states, decisions, efforts, and outputs.

NECESSARY AND SUFFICIENT CONDITIONS

1. If σ^* is a self-enforcing relational contract, then there exists a reward scheme B^* that is credible in σ^*.
    - IC is immediate.
    - $B_i^* \geq \overline{U}_i$ or else agent would walk away.
    - $B_i^* \leq \delta E[S_{i,t+1}|y_{i,t}]$ or else principal would walk away from i

2. If σ is a relational contract with a credible reward scheme B, then there is a self-enforcing relational contract σ^* inducing same joint distribution over states, decisions, efforts, and outputs.

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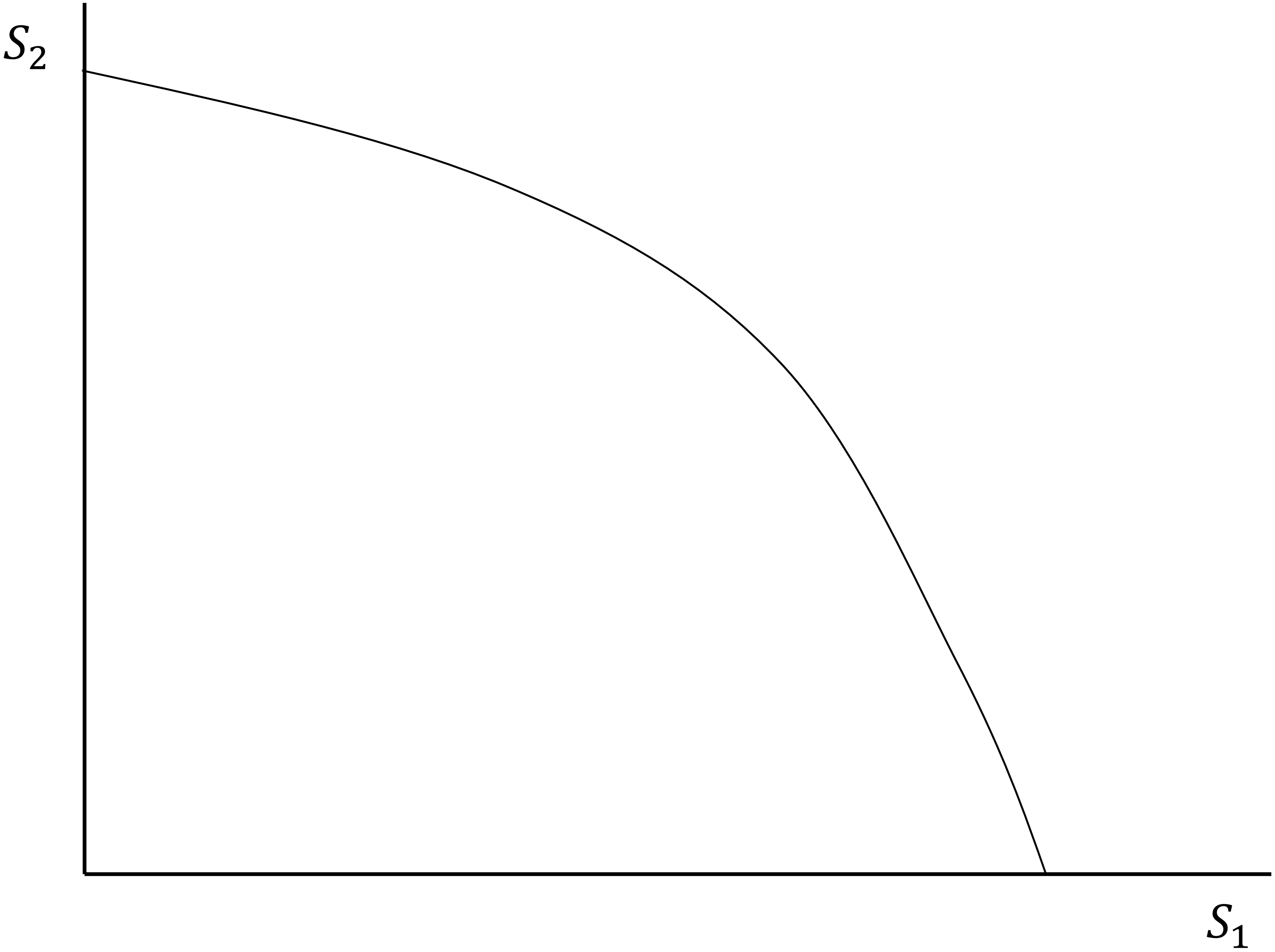
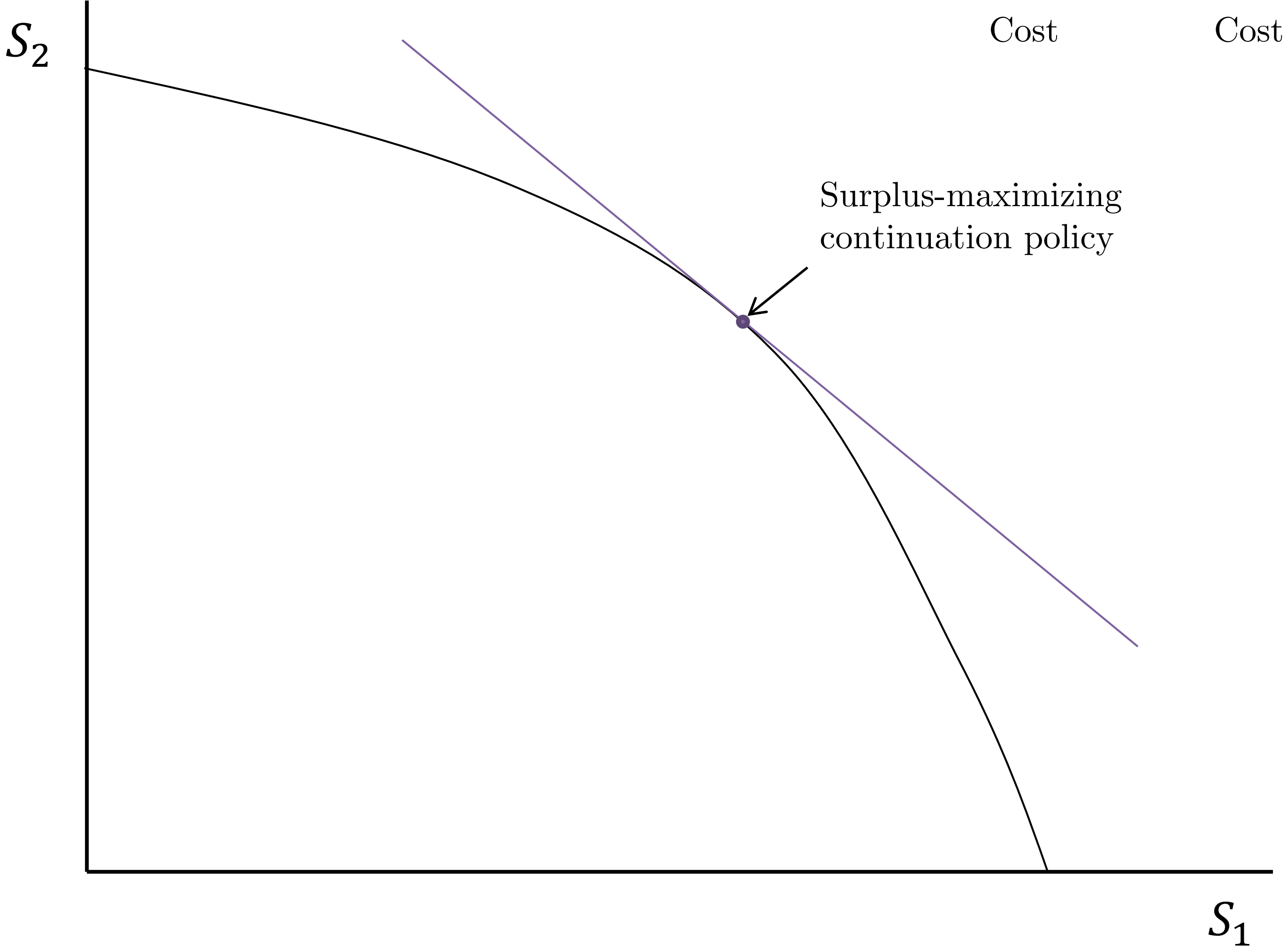
2. If σ is a relational contract with a credible reward scheme B, then there is a self-enforcing relational contract σ^* inducing same joint distribution over states, decisions, efforts, and outputs.
   - Need to get principal to choose policy and agents to choose efforts in σ
   - Transfer expected surplus to agents via wages: principal willing to choose policy
   - Output-contingent fines set to give agent B after paying them: agents willing to pay these fines, since B is credible, and willing to choose efforts

BACKWARD-LOOKING POLICIES IN SMOOTH GAMES

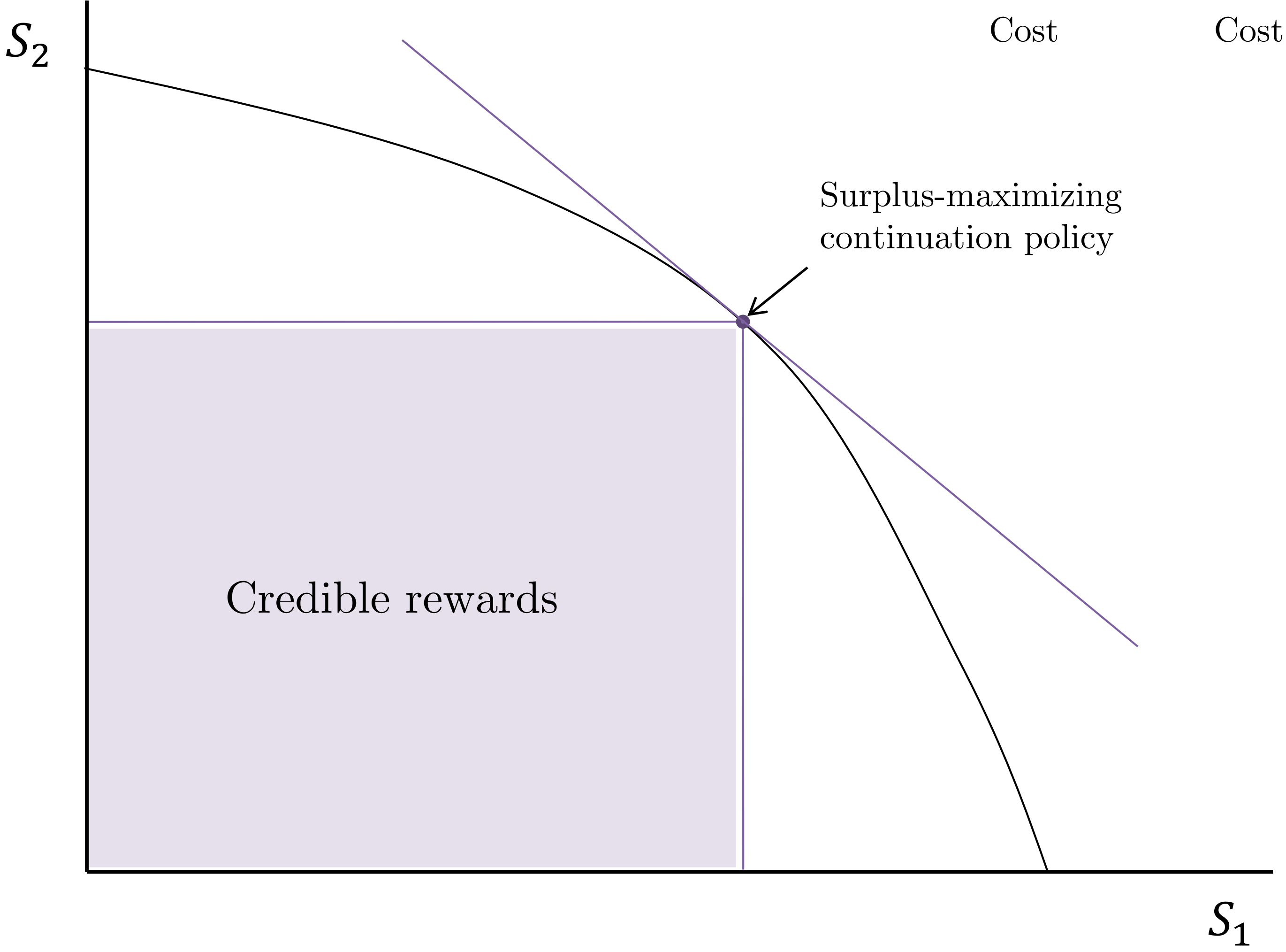


BACKWARD-LOOKING POLICIES IN SMOOTH GAMES

Direct Cost	Incentive Cost	Incentive Benefit

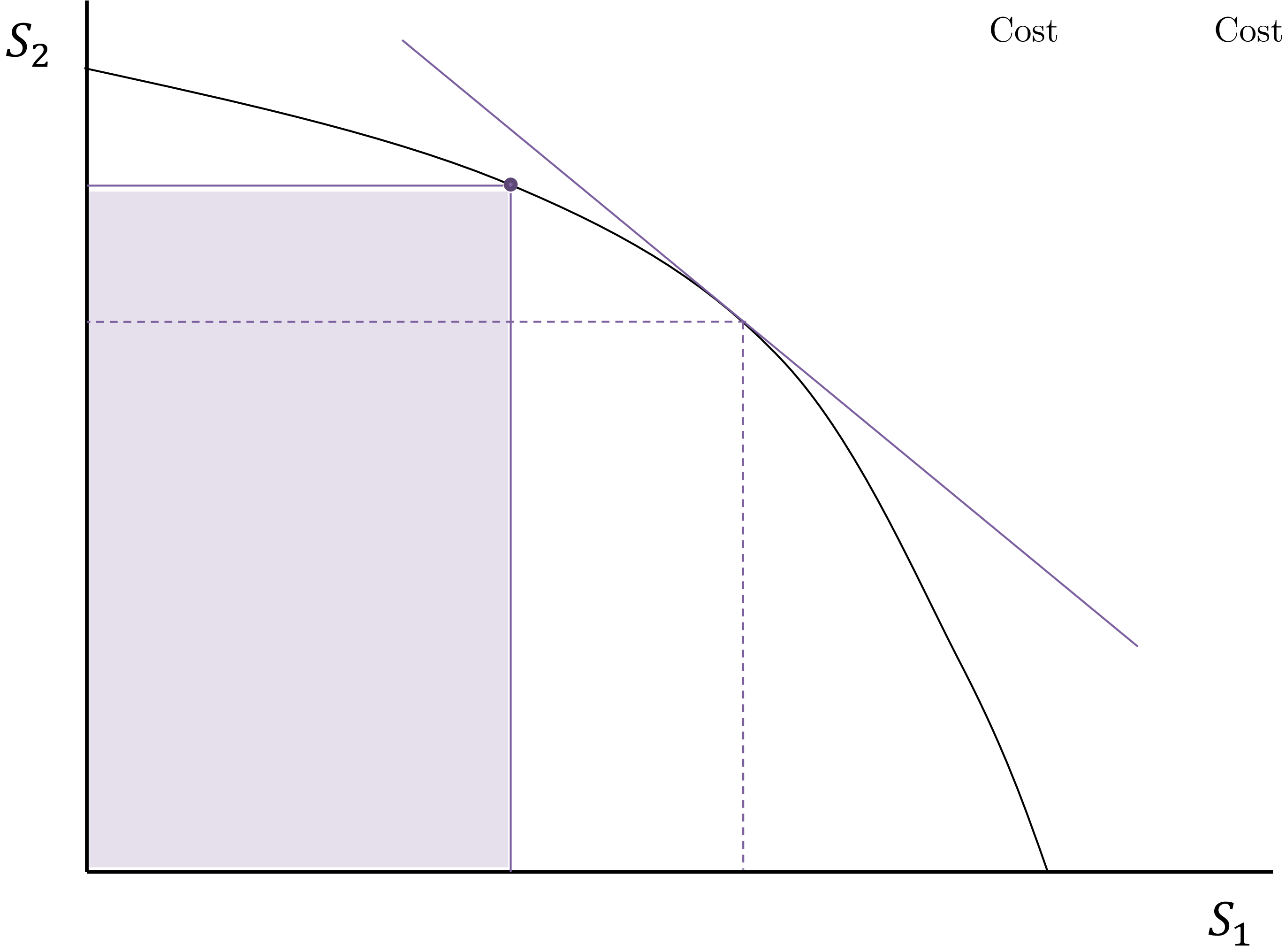


BACKWARD-LOOKING POLICIES IN SMOOTH GAMES



Direct Cost	Incentive Cost	Incentive Benefit

Backward-looking Policies in Smooth Games

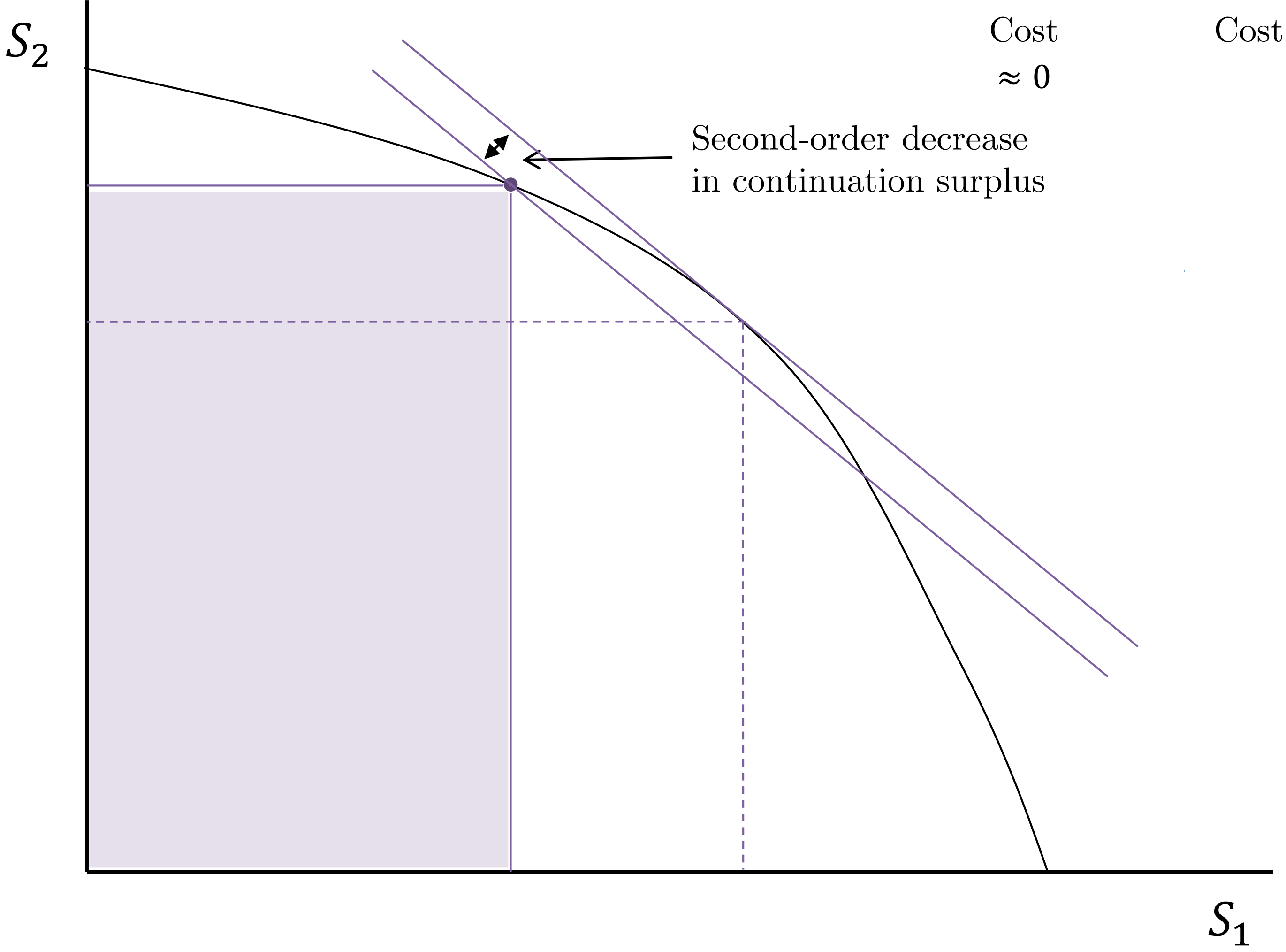


Direct Cost

Incentive Cost

Incentive Benefit

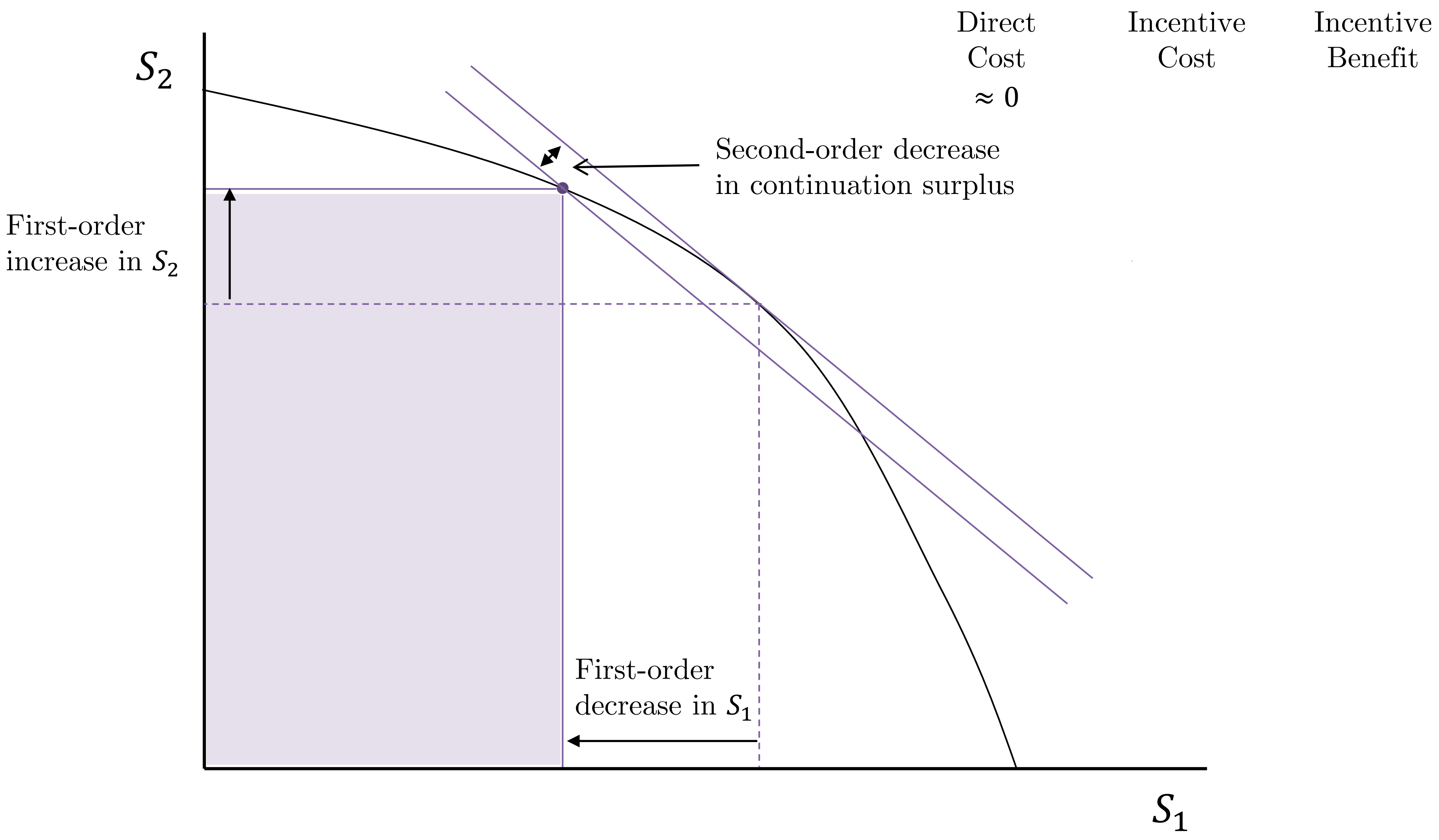
BACKWARD-LOOKING POLICIES IN SMOOTH GAMES



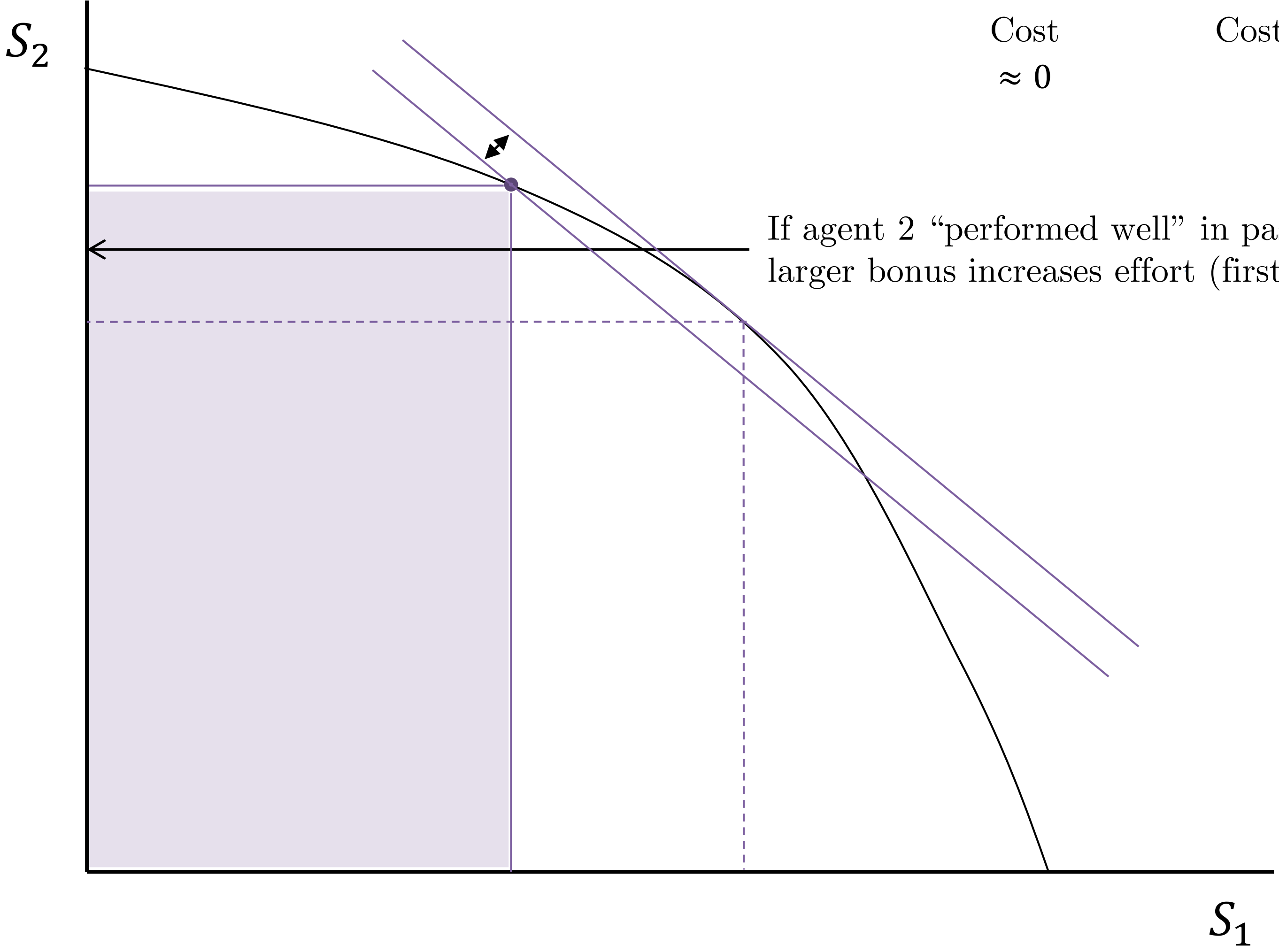
Second-order decrease in continuation surplus

Direct Cost	Incentive Cost	Incentive Benefit
≈ 0		

BACKWARD-LOOKING POLICIES IN SMOOTH GAMES



BACKWARD-LOOKING POLICIES IN SMOOTH GAMES

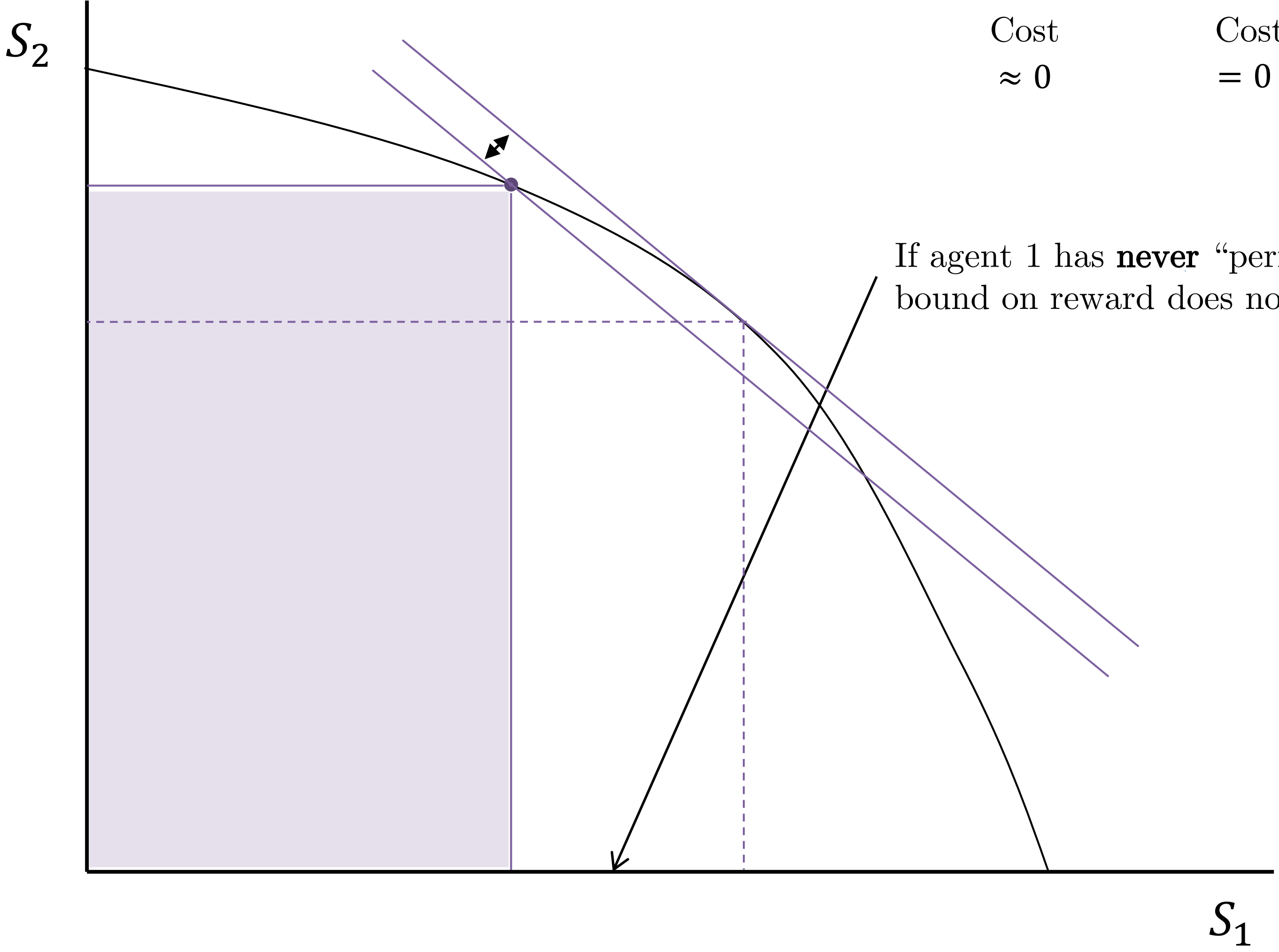


Direct Cost	Incentive Cost	Incentive Benefit
≈ 0		> 0

If agent 2 “performed well” in past, larger bonus increases effort (first-order)…

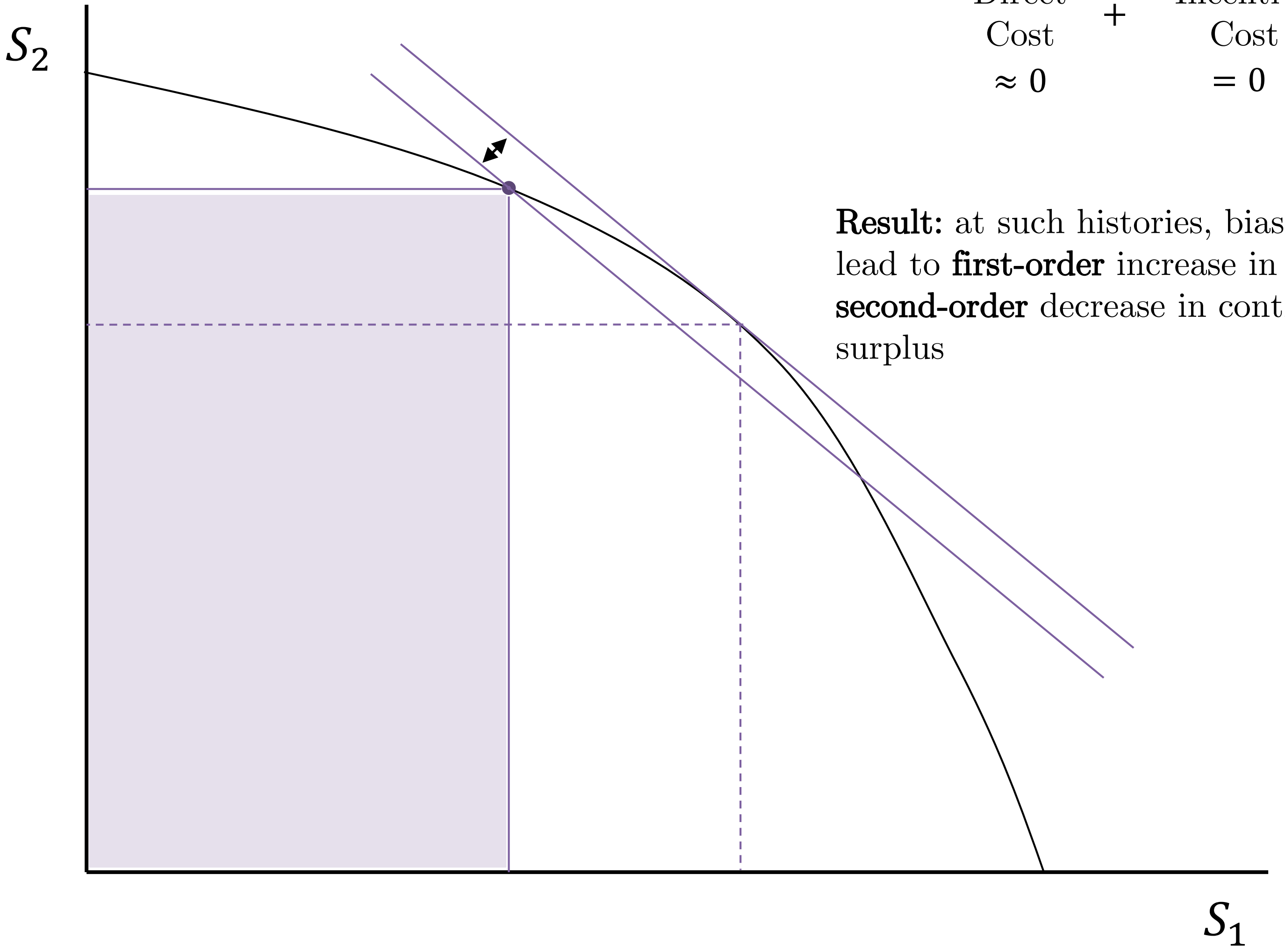
BACKWARD-LOOKING POLICIES IN SMOOTH GAMES

Direct Cost	Incentive Cost	Incentive Benefit
≈ 0	$= 0$	> 0



If agent 1 has **never** “performed well,” upper bound on reward does not bind

BACKWARD-LOOKING POLICIES IN SMOOTH GAMES



$$\begin{array}{ccccc} \text{Direct} & & \text{Incentive} & & \text{Incentive} \\ \text{Cost} & + & \text{Cost} & < & \text{Benefit} \\ \approx 0 & & = 0 & & > 0 \end{array}$$

Result: at such histories, biased decisions lead to **first-order** increase in effort, **second-order** decrease in continuation surplus

TWO COMPLICATIONS

1. Providing conditions on the primitives that ensure the frontier between any two agents is differentiable.

2. With $N \geq 2$, not obvious that incentive cost is zero, as other agents' dynamic enforcement constraints might bind

SMOOTH MEAN-SHIFTING GAMES

1. Decisions are weights $d_{i,t} \geq 0$ assigned to each agent $\left(\sum_i d_{i,t} \leq 1\right)$
2. States of the world θ_t are i.i.d.
3. Outside options $\bar{u}_i$ depend only on states of the world
4. Effort costs $c(\cdot)$ are smooth, strictly increasing, and strictly convex
5. Output distributions: P_i^H FOSD P_i^L and

$$P_i(y_i|\theta, d, e_i) = (1 - e_i)P_i^L(y_i - \gamma_i(\theta, d)) + eP_i^H(y_i - \gamma_i(\theta, d))$$

MAIN RESULT

Define: $e_i^{FB}(d_i, \theta) = \arg\max E[y_i | d_i, \theta, e_i] - c(e_i)$

In a smooth mean-shifting game, let σ^* be a surplus-maximizing recursive equilibrium

For agents i, j, consider a history h_0^{t+1} such that:

1. Agent i chooses positive effort less than e_i^{FB} in t
2. Agent i's output had strictly positive score in t
3. Agent j's output had weakly negative score for all $t' \leq t$
4. Both i and j have positive weight $(d_{i,t}, d_{j,t} > 0)$

Result: for almost all such h_0^{t+1}, $\sigma^* | h_0^{t+1}$ is not surplus-maximizing

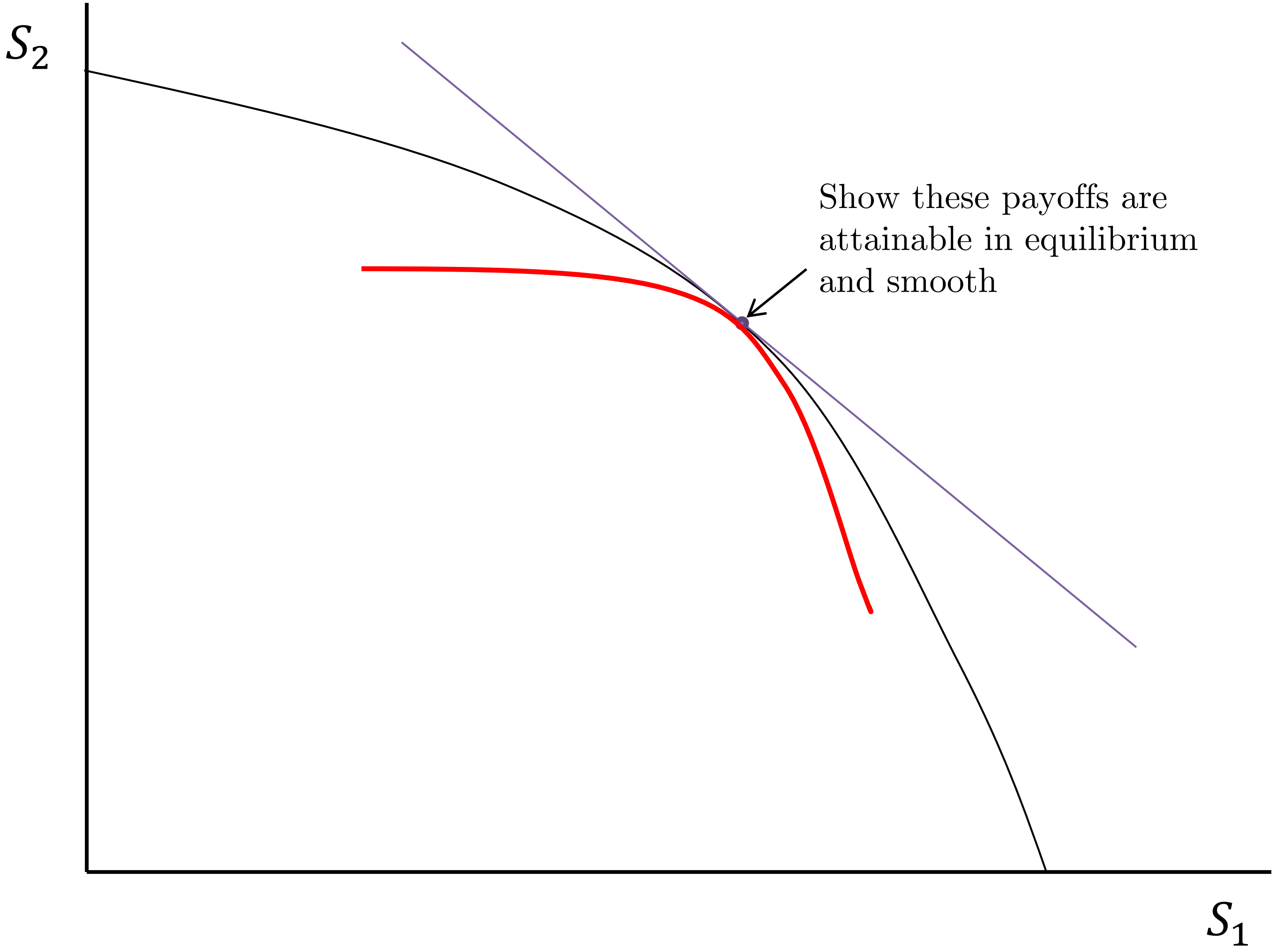
MAIN RESULT

Define: $e_i^{FB}(d_i, \theta) = \arg\max E[y_i | d_i, \theta, e_i] - c(e_i)$

In a smooth mean-shifting game, let σ^* be a surplus-maximizing recursive equilibrium

Result: Consider a smooth mean-shifting game. Suppose that $\lim_{d_i \to 0} \partial \gamma_i / \partial d_i = \infty$, $\min_{e_i} c'(e_i) = 0$ for all i. Then there are $\delta_L < \delta_H$ such that for all $\delta \in [\delta_L, \delta_H]$, no surplus-maximizing relational contract is sequentially surplus-maximizing.

Why is Dyad-Surplus Frontier Smooth?



Show these payoffs are attainable in equilibrium and smooth

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HIRING MODEL

Decision: how many workers to hire in each period: $D_t = \{1,2\}$

State: demand, growing, persistent. $\Theta = \{W, R\}$ with $0 < W < R$

- If demand is weak, it becomes robust with probability ρ
- If demand is robust, it remains robust

Binary effort: $e_{i,t} \in \{0,1\}$ at cost $ce_{i,t}$

Per-worker productivity falls in number of workers hired:

- $y_{i,t} = \theta_t e_{i,t}$ if $d_{i,t} = 1$ and $y_{i,t} = \theta_t \alpha e_{i,t}$ if $d_{i,t} = 2$, $\alpha < 1$

DELAYED GROWTH

Assume:

1. In first-best, should hire one when demand weak, two when robust
2. Dyad-surplus larger when demand is robust

There exist $\delta_L < \delta_H$ such that for $\delta \in (\delta_L, \delta_H)$, any surplus-maximizing relational contract satisfies:

1. If $\theta_0 = R$, then $d_t = 2$ in every period t
2. If $\theta_0 = W$, then $d_t = 1$ whenever $\theta_t = W$. Moreover, there exists $t' > 0$ such that $\Pr[d_{t'} = 1, \theta_{t'} = G] > 0$

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PERMANENT INVESTMENT

Decision = one-time, permanent investment in one agent

- Investment increases agent output for fixed effort
- Agents have differing returns from investment
- Moral hazard: output is stochastic

Result: award investment in a tournament

- Distort investment: if low-return agent performs well, gets investment
- Agent with investment produces more in future, so can be promised larger reward

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  - Public Monitoring
  - Imperfect Coordination

SEQUENTIALLY SURPLUS-MAXIMIZING PBE

Definition: Let $\bar{V} = \max_{\sigma^* | \sigma^* \in PBE} E_{\sigma^*}[\sum_i S_{i,0}]$. Then a PBE is a **sequentially surplus-maximizing PBE** if in each $t \geq 0$, $\bar{V} = E_{\sigma^*}[\sum_i S_{i,t}]$.

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- Seq. surplus-max PBE → $\frac{\partial\gamma_i}{\partial d_i} = \frac{\partial\gamma_j}{\partial d_j}$ for all i and j, so d_t^* is uniquely determined
- Seq. surplus-max PBE → Seq. surplus-max RE
- But surplus-max RE is not seq. surplus-max, so neither is surplus-max PBE

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PUBLIC MONITORING

Suppose all variables (except effort) publicly observed

Biased decisions decrease total continuation surplus

Result: if monitoring is imperfect but public, then any surplus-maximizing relational contract is sequentially surplus-maximizing

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IMPERFECTLY COORDINATED PUNISHMENT

Modification of hiring application: deviations are ϵ-private

- The first time i chooses $a_{i,t} = 0$, all agents observe this with probability $1 - \epsilon$
- Otherwise, only principal observes it. Subsequent $a_{i,t} = 0$ only observed by principal.

Result: if $\epsilon > 0$, there is an open set of parameters under which no surplus-maximizing rel. con. is sequentially surplus-maximizing

- If principal reneges on i, all agents observe subsequent rejection w/prob $1 - \epsilon$ and punish, destroying total surplus $\delta E[\sum_{i \leq N} S_{i,t+1}]$
- Otherwise, only i punishes principal, destroying surplus $\delta E[S_{i,t+1}]$
- i's dyad-surplus looms larger for principal than j's dyad-surplus

RELATED LITERATURE

Sequentially Efficient Relational Contracts

Bull (1987); MacLeod and Malcomson (1989); Baker, Gibbons, and Murphy (1994); Levin (2002, 2003)

Sequential Inefficiencies and Dynamics in Formal Contracts

Fudenberg, Holmstrom, and Milgrom (1990)

Sequential Inefficiencies and Dynamics in Relational Contracts

Persistent private information: Levin (2003); Fuchs (2007); Halac (2012)

Limited transfers: Board (2011); Fong and Li (2017); Li, Matouschek, and Powell (2017); Ke, Li, and Powell (2018)

Limited multilateral enforcement: Ali, Miller, and Yang (2016); Andrews and Barron (2016)

CONCLUSION

Flexible framework of backward-looking policies in relational contracts

- Decisions make past promises credible, rather than maximizing future surplus

Biases important for broad class of games

- If (and only if) agents cannot coordinate punishments
- Relational contracts evolve in history-dependent ways

Biases manifest in realistic ways

- Lagged hiring, delayed investment

EXTRA SLIDES

COSTS AND BENEFITS OF BIASED DECISIONS

Direct Cost	Incentive Cost	Incentive Benefit

Costs and Benefits of Biased Decisions

$$\begin{array}{c} \text{Direct} \\ \text{Cost} \end{array} + \begin{array}{c} \text{Incentive} \\ \text{Cost} \end{array} < \begin{array}{c} \text{Incentive} \\ \text{Benefit} \end{array}$$

Costs and Benefits of Biased Decisions

$$\begin{array}{ccccc} \text{Direct} & & \text{Incentive} & & \text{Incentive} \\ \text{Cost} & + & \text{Cost} & < & \text{Benefit} \\ \approx 0 & & = 0 & & > 0 \end{array}$$

CONTRIBUTIONS

General model of policies in relational contracts

Biased decisions optimal among recursive equilibria in a class of games

Show that equilibrium refinement does not drive result

Applications to hiring lags and distorted investments

SMOOTH GAMES

1. S_i frontier is downward-sloping

- **Decisions** are weights $d_{i,t} \geq 0$ assigned to each agent $\left(\sum_i d_{i,t} \leq 1\right)$
- **Higher $\boldsymbol{d_i}$** means: higher expected y_i (strictly concave) that is (weakly) more informative of effort (effort-independent garbling). No effect on y_{-i}.

2. S_i frontier is smooth

- **States of the world $\boldsymbol{\theta}$** are independent of past decisions
- **Outside options** $\bar{u}_i$ depend only on states of the world
- **Effort** costs $c(\cdot)$ are smooth, strictly increasing, and strictly convex

3. Changing one agent's effort can affect others' incentives

- **Output distributions $\boldsymbol{P_i}$** are smooth and satisfy Mirrlees-Rogerson conditions

Smooth Games (Formal)

A game is **smooth** if…

- For every t, $D_t = \{(d_1, \dots, d_N) | d_i \geq 0, \sum_i d_i \leq 1\}$ and θ_t is iid
- Outside options depend only on θ_t
- Effort costs $c(\cdot)$ are smooth, strictly increasing, and strictly convex
- P_i depends only on d_i, θ, e_i; is smooth in all arguments with density p_i; has full support; is strictly MLRP-increasing in e_i; and satisfies CDFC
- Expected output $E[y_i | d_i, \theta, e_i]$ is strictly increasing and strictly concave in $\{d_i, e_i\}$
- Higher decisions are more informative: if $d_i \geq \widetilde{d}_i$, then there exists an effort-independent garbling $R(x_i | y_i)$ with density r_i such that

$$\int_{y_i \leq \overline{y_i}} p_i(y_i | \theta, \tilde{d}_i, e_i) dy_i = \int_{y_i \leq \bar{y}_i} r_i(x | y_i) p_i(y_i | \theta, d_i, e_i) dy_i$$

STATEMENT OF MAIN RESULT (FORMAL)

Define: $e_i^{FB}(d_i, \theta) = \underset{e_i}{\arg\max}\, E[y_i | d_i, \theta, e_i] - c(e_i)$

In a smooth game, let σ^* be a surplus-maximizing recursive equilibrium

For agents i, j, let E_t be a set of histories h_0^{t+1} such that:

1. $e_{i,t} > 0$ but $e_{i,t} < e_i^{FB}(d_i, \theta)$
2. $\frac{\partial p_i / \partial e_i}{p_i}\left(y_{i,t} \middle| d_{i,t}, \theta_t, e_{i,t}\right) > 0$
3. $\frac{\partial p_j / \partial e_j}{p_j}\left(y_{j,t'} \middle| d_{j,t'}, \theta_{t'}, e_{j,t'}\right) \leq 0$ for all $t' \leq t$
4. $d_{i,t+1} < 1$ and $d_{j,t+1} > 0$ with positive probability

Result: for almost every $\mathrm{h}_0^{t+1} \in E_t$, $\sigma^* | h_0^{t+1}$ is not surplus-maximizing

SMOOTH MEAN-SHIFTING GAMES

A smooth game is a **smooth mean-shifting game** if and θ_t are i.i.d. and:

$$P_i(y_i|\theta, d, e_i) = (1 - e_i)P_i^L(y_i - \gamma_i(\theta, d)) + eP_i^H(y_i - \gamma_i(\theta, d))$$

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