Markets for Rhetorical Services

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Abstract

We study markets for rhetorical services, which improve the purchaser's ability to make compelling arguments to a third party and include the services offered by public relations consultants, advertising agencies, and lawyers. An agency privately sells rhetorical servers to a sender, the purchase of which plays a "signal-jamming" role by secretly altering the distribution of a signal observed by a receiver. The receiver's beliefs about purchase decisions therefore influence demand for rhetorical services, which in turn shapes profit-maximizing behavior and the welfare of both the sender and receiver. We use our framework to analyze optimal pricing, the effects of competition, and how regulation changes market outcomes.

1 Introduction

Crafting compelling arguments is hard work. Doing so on another's behalf, and doing it well, is big business. Each year, clients spend billions hiring lawyers to develop legal arguments, public relations firms to write speeches and press releases, agencies to create advertising campaigns, and political consultants to weave narratives and spin the day's news. Agencies that sell these types of services command a premium because they understand how to make an argument more persuasive to its intended audience.

In some situations, the most compelling argument is one that deftly navigates the gray area between truth and lie. Oreskes and Conway (2011) tells the story of a small group of scientists who were hired to cast doubt on mounting evidence about the dangers of smoking and the reality of climate change. With a veneer of scientific authority, these so-called merchants of doubt "sold a plausible story about scientific debate.... A reasonable journalist, not to mention an ordinary citizen, could be forgiven for having been fooled by it." (p. 245). Not all arguments aim to mislead, however; sometimes the most compelling argument strips away all the distractions and makes a simple point clearly. Lief et al. (1999) emphasizes this point in describing William Kunstler's closing argument in the trial of the "Chicago Seven": "Kunstler—like all great attorneys—quickly established his central theme, referring back to it often. Kunstler recognized that jurors can only focus on limited amounts of information and it was his job to identify the main issue." (p. 109)

We propose a framework for studying markets for *rhetorical services*—services aimed at improving a client's ability to make compelling arguments to a third party. Understanding how these markets function is important because, as McCloskey and Klamer (1995) argues and the examples above illustrate, transactions involving rhetorical services constitute a sizeable fraction of the economy. Moreover, rhetorical services are not like other goods and services. Their value arises from their ability to craft more persuasive arguments and therefore depends on how the intended audience interprets those arguments. Finally, these services influence the audience's ability to make informed decisions, creating scope for

regulation that can improve the operation of these markets.

The early days of the U.S. advertising industry paint a vivid example of how audience beliefs shape demand for rhetorical services. As Wu (2016) recounts, the first mass-market advertising campaign consisted of mailed pamphlets for a "Nerve Tonic" sold by Dr. Shoop's Restorative. These pamphlets included many outright lies but nevertheless initially boosted demand for the tonic, so much so that many other patent medicine companies adopted similar marketing tactics. This rising flood of advertising met concomitant increases in customer skepticism, however, as potential customers learned that so-called "snake-oil salesmen" could make fantastical claims regardless of the quality of their products. The resulting backlash ultimately undermined demand for advertising services, as the patent medicine boom gave way to a decades-long collapse of the market for advertisers. While demand recovered after the success of World War I propaganda and the passage of early truth-in-advertising laws, this example illustrates a more general feedback loop: as customers adapt to new advertising techniques, the value of those techniques, and the prices they command on the market, change as well.

We analyze an elemental model in which a sender would like to persuade a receiver that a binary state is high. The receiver observes a public signal—say, an advertisement—that is informative about this state. This signal is drawn from some fixed distribution if the sender does not intervene. However, by privately purchasing rhetorical services from an agency— say, by hiring an outside advertising firm rather than designing an ad on her own—the sender can change the signal distribution to some other fixed distribution. The receiver does not observe either the price of the rhetorical service or the sender's purchase decision. We therefore represent sender-receiver communication as a mapping from the state to a signal distribution, a formulation that we argue captures "persuasive communication," in which the sender's arguments are more than simply empty messages but less than conclusive proof. Rhetorical services then serve a "signal-jamming" role by secretly improving the sender's arguments and so changing the mapping from the state to the signal distribution (Fudenberg

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and Tirole (1986), Holmstrom (1999), Prat (2005), Ottaviani and Sorensen (2006a,b)).

While simple, we use this model to highlight two contrasting types of rhetorical services that enable stronger arguments in different states of the world. Our first results characterize demand for these two types of services. A service *embellishes* if it increases the signal only if the state of the world is low. Such services include the efforts of the "merchants of doubt" and other techniques that obfuscate the defects or overstate the merits of a product, candidate, or cause. The sender values embellishing services because they increase the receiver's posterior belief if the state is low. As with the example of Dr. Shoop's Restorative, we show that embellishing services encourage skepticism and so undermine their own demand in equilibrium: if the receiver believes that such services are widely used, then he puts little weight on the signals that those services make likely, which reduces sender willingness-to-pay. We also show that embellishing services harm the receiver by decreasing his information in equilibrium.

The opposite of an embellishing service is a *clarifying* service, which increases the signal only if the state is high. Such services include compelling legal arguments, policy speeches, and scholarly works, all of which succeed by providing clear, catchy, and memorable ways to convey real information to the receiver. The sender values clarifying services because they increase the receiver's posterior belief if the state is high. We show that clarifying services create their own demand in equilibrium: if the receiver believes such services to be widely used, he assigns a high posterior to the signals that they make likely, which increases sender willingness-to-pay. Such services can therefore exhibit upward-sloping equilibrium demand, so that a single price is associated with multiple market outcomes (a feature shared by other settings with belief spillovers, such as Akerlof et al. (2018)'s analysis of markets with network externalities). Clarifying services also make the signal more informative and so benefit the receiver.

Next, we turn to the supply of rhetorical services and show why it is important to model these services differently from other products. A profit-maximizing producer of rhetorical

services neglects two important spillovers when setting its price. First, the receiver does not directly participate in the market for rhetorical services and so equilibrium prices do not reflect his welfare. Second, as the receiver does not observe market outcomes, the agency cannot influence his beliefs by adjusting its price. We show that market outcomes are not necessarily efficient due to these two spillovers. For instance, competition among agencies does not necessarily benefit the receiver or even the sender. We also show that these spillovers are exacerbated if the agency can privately design its rhetorical service, since in equilibrium, its chosen design leads to extreme receiver skepticism, the total failure of information transmission, low receiver welfare, and potentially low agency profits.

These market failures suggest that regulatory interventions might be valuable in markets for rhetorical services. Regulators have access to unique instruments in these markets, since they can employ public education campaigns or truth-in-advertising laws to essentially change the information content of the signal. We model these interventions as decreasing the signal in the low state and thereby improving the receiver's information and welfare. However, these interventions also change demand in the market for rhetorical services, since they decrease receiver skepticism and so make strong arguments all the more valuable. Depending on the nature of the rhetorical service, this change in demand can either compound or undermine the positive welfare effects of regulation.

Finally, we develop a sender-receiver model in order to better understand the nature of the signal observed by the receiver. This model introduces the idea that arguments, even arguments in support of the same claim, may have different strengths. We posit that (i) stronger arguments are easier to make if they are true; (ii) receivers can tell whether an argument is strong or weak but cannot otherwise observe its veracity;[1] and (iii) the sender has private information, not only about the state of the world, but also about her *rhetorical ability* and hence how strong her arguments can be.[2] In a receiver-optimal equilibrium,

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[1] We do not analyze *why* receivers cannot perfectly tell whether an argument is true or false. One could imagine that they engage in coarse reasoning, are attention constrained, or have other cognitive limits (see, e.g., Mullainathan et al. (2008)).

[2] Our notion of rhetorical ability is related to but distinct from Dewan and Myatt (2008)'s notion of the

stronger arguments are more convincing, so the sender always makes the strongest possible argument in favor of her preferred state. The resulting distribution over arguments replicates the signal distribution from the market for rhetorical services. An agency then sells access to stronger arguments, where it clarifies or embellishes by enabling stronger truthful or false arguments, respectively.

Related Literature

The closest papers to our setting study markets for information disclosure (see Dranove and Jin (2010) for a survey). For example, Lizzeri (1999) studies the optimal design of a certification, which is essentially a public experiment that a firm can publicly buy. Our study complements this analysis by focusing on a different mechanism—signal-jamming rather than signaling—and considering demand for services that have a fixed effect on the signal distribution. A related literature, including Calzolari and Pavan (2006a,b) and Dworczak (2018), studies mechanism design problems in which the designer can disclose information to third parties. More broadly, a variety of papers study information disclosure, either as a way to persuade a buyer (Ottaviani and Prat (2001); Johnson and Myatt (2006); Eso and Szentes (2007)), or as a way to inform one's own decision-making (Admati and Pfleiderer (1986, 1990); Horner and Skrzypacz (2016); Bergemann et al. (2018)). Kamenica and Gentzkow (2011) is more distantly related, as we focus on a market transaction that is private and has a fixed effect on a signal distribution.

A vast literature focuses on the "downstream" effects of rhetorical services. Many of these papers study the advertising industry (see Bagwell (2007) for a survey) through the lens of a variety of models both classic (Stigler (1961); Butters (1977); Grossman and Shapiro (1984); Nelson (1974); Milgrom and Roberts (1986a)) and contemporary (Johnson and Myatt (2006); Mullainathan et al. (2008)). A more recent theoretical literature has focused on the regulation of false or misleading advertising (Dellarocas (2006); Rhodes and Wilson (2017)).

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clarity of communication. In our setting, a rhetorically gifted sender is able to both clearly argue if her preferred state is realized and effectively dissemble if it is not.

We differ from these papers by studying the upstream market for *advertising agencies*. Other papers have studied other input markets for advertising; for example, Bergemann and Bonatti (2011) analyzes the value of better ad targeting, Edelman et al. (2007) and others study auctions for the ad space itself, and Fainmesser and Galeotti (2018) study the market for influencers who can widely and credibly recommend products. While receivers in our model are Bayesian, the key assumption is that they interpret advertisements based in part on their beliefs about how those advertisements were created. Our mechanism therefore does not preclude the idea that advertising may also exploit behavioral biases.

In addition to advertising, rhetorical services are widely employed in politics, law, and the media; see DellaVigna and Gentzkow (2010) for a survey. Within these literatures, papers have focused on the interaction between persuasion and market concentration (Mullainathan and Shleifer (2005); Prat (2017)), the interaction between media and the government (Besley and Prat (2006)), or political persuasion in the presence of bias (Murphy and Shleifer (2004)). Again, we differ from these papers by studying the effects of a particular input market—the market for arguments—on equilibrium persuasion.

As we will show, the economics of these markets is related to markets for both network and status goods. Like network goods (Farrell and Klemperer (2007)), demand for rhetorical services depends on beliefs about quantity, and as in Akerlof et al. (2018), these belief spillovers can lead to upward-sloping equilibrium demand. Unlike network goods, however, the value of a rhetorical service varies with the beliefs of a third party who does not observe prices. Like status goods (Bagwell and Bernheim (1996); Rayo (2013)), the value of a rhetorical service depends on a third party's beliefs, though the purchase of rhetorical services itself is private and so does not directly signal information.

Rhetorical services serve a "signal jamming" role in our model (Fudenberg and Tirole (1986); Dewatripont et al. (1999); Holmstrom (1999)), in the sense that the sender's purchase decision is private and affects the distribution of a signal observed by the receiver. We model these services in an abstract way so that we can emphasize types of signal jamming that

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are not studied elsewhere. We particularly focus on "embellishing" and "clarifying" services, which alter the signal distribution in different states and so have contrasting implications for demand, welfare, and regulation.

2   Model

Consider a **sender** ("she") who wants to persuade a **receiver** ("he") that a binary state of the world is high. The receiver observes a signal that is informative about the underlying state. The sender can alter the distribution of this signal by privately choosing to purchase a rhetorical service that is produced and sold by an **agency** ("it"). After observing this signal, the receiver makes a decision that affects both his and the sender's payoff; his ideal decision is increasing in his posterior belief that the state is high. The sender's willingness-to-pay for the rhetorical service depends on her type, which determines her returns from convincing the receiver to make a higher decision. While we mostly consider a monopolist agency, Section 4 also analyzes undifferentiated Bertrand competition among agencies.

Formally, we analyze a game with the following timing:

1. The sender privately observes her type $t \in \mathbb{R}$, which is drawn from a uniform distribution on $[0, 1]$.[3]

2. The agency chooses a price $p \geqslant 0$, which is observed by the sender but not the receiver.[4]

3. The sender chooses whether or not to purchase the rhetorical service, $x \in \{0, 1\}$, which is observed by the agency but not the receiver.

4. A binary state of the world $\omega \in \{0, 1\}$ is realized but not observed, with $\Pr\{\omega = 1\} = \gamma \in (0, 1)$.

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[3]We assume that a sender's payoff is monotone and exhibits strictly increasing differences in her type, but we make no assumptions about how her type affects the curvature of her payoff. Therefore, the assumption that it is distributed uniformly is a normalization.

[4]We do not allow the agency to offer menus of contracts, but this is without loss: the agency cannot benefit from offering a menu because the probability of sale enters linearly in the sender's utility.

5. The receiver observes a signal $s \in \mathbb{R}$, where $s \sim F_\omega(\cdot)$ if $x = 0$ and $s \sim G_\omega(\cdot)$ if $x = 1$.

6. The receiver makes a decision $d \in \mathbb{R}$.

The agency's, sender's, and receiver's payoffs are respectively $\pi = (p-c)x$, $U_S = u_S(d,t) - px$, and $U_R = u_R(d,\omega)$, where $c \geqslant 0$ is the cost of providing the rhetorical service. We assume that payoffs are smooth, $u_S(\cdot, t)$ is strictly increasing for any $t > 0$, and $u_S(\cdot)$ and $u_R(\cdot)$ exhibit strictly increasing differences in (d,t) and (d,ω), respectively. That is, the sender's payoff is strictly increasing in the receiver's decision, the receiver's decision is strictly increasing in his posterior belief that $\omega = 1$, and the marginal return to the sender from inducing a higher posterior belief is increasing in t. While our analysis would mostly extend to a degenerate distribution over t, assuming a non-degenerate distribution generates a well-behaved demand curve for the rhetorical service. The interpretation of t depends on the context; in a legal setting, for instance, t might correspond to a defendant's legal exposure. Our solution concept is (weak) Perfect Bayesian Equilibrium.

We require $F_\omega(\cdot)$ and $G_\omega(\cdot)$ to have densities and common supports.

Assumption 1 *The distribution functions $G_\omega(\cdot)$ and $F_\omega(\cdot)$ have continuously differentiable densities $g_\omega(\cdot)$ and $f_\omega(\cdot)$ defined on a common support.*

The agency, which depending on the application could be an advertising firm, pundit, speechwriter, or legal team, sells a service that changes the distribution over signals from $F_\omega(\cdot)$ to $G_\omega(\cdot)$. These distributions are an abstract way to capture many different models of sender-receiver communication, since the equilibrium outcomes of such models take the form of mappings from the state to a message or some other type of signal. In particular, both cheap talk and verifiable disclosure correspond to different limiting cases. Since the sender wants to induce the highest possible posterior, cheap talk would result in uninformative communication, represented by $F_1(\cdot) = F_0(\cdot)$. The rhetorical service could then be interpreted as generating evidence of $\omega = 1$ that is difficult to fake. Similarly, (costless) verifiable disclosure would lead to unraveling, which would be represented by assuming that $F_0(\cdot)$ and $F_1(\cdot)$ had

non-overlapping supports. In that case, the rhetorical service might create "fake" evidence when $\omega = 0$.

The following assumption ensures that regardless of the sender's purchase behavior, higher signals provide stronger evidence that $\omega = 1$. We also assume that $g_\omega(\cdot)$ single-crosses $f_\omega(\cdot)$ from below, which implies that purchasing the rhetorical service tends to increase the signal.

Assumption 2 *The likelihood ratios $\frac{f_1(\cdot)}{f_0(\cdot)}$, $\frac{g_1(\cdot)}{g_0(\cdot)}$, $\frac{f_1(\cdot)}{g_0(\cdot)}$, and $\frac{g_1(\cdot)}{f_0(\cdot)}$ are increasing, with $\frac{f_1(\cdot)}{f_0(\cdot)}$ and $\frac{g_1(\cdot)}{g_0(\cdot)}$ strictly so. The rhetorical service leads to higher signals: $G_\omega \neq F_\omega$ for at least one $\omega \in \{0, 1\}$, and $g_\omega(\cdot) - f_\omega(\cdot)$ single-crosses 0 from below for each $\omega \in \{0, 1\}$.*

We maintain Assumptions 1 and 2 throughout the analysis. They are satisfied if, for instance, $F_1(\cdot)$, $F_0(\cdot)$, $G_1(\cdot)$, and $G_0(\cdot)$ are exponential distributions with respective parameters λ_1^F, λ_0^F, λ_1^G, and λ_0^G that satisfy $\lambda_1^G \leqslant \lambda_1^F \leqslant \lambda_0^G \leqslant \lambda_0^F$, $\lambda_1^F < \lambda_0^F$, and $\lambda_1^G < \lambda_0^G$.

We assume that the sender does not know the state when she makes her purchase decision. While not essential for our basic intuition, we believe that this assumption is natural in many settings. If the sender is a defendant in a court case, for example, γ represents her beliefs about her guilt when she decides which law firm should represent her, while ω represents the extent of her legal liability and is revealed only after she retains counsel. Similarly, in advertising, ω might be the actual value of the product to a customer, in which case γ is the probability that that product is a "home run" when its advertising campaign is designed.

More importantly, we assume that the receiver can observe neither the price nor the sender's purchase decision. This assumption is natural if the sender tries to persuade the receiver even if she does not purchase rhetorical services. For instance, a firm might hire an outside agency to design an advertising campaign (represented by distribution $G_\omega(\cdot)$) or rely on its less expensive, but potentially less effective, in-house team (represented by $F_\omega(\cdot)$). In this example, the advertisement itself is represented by s and is observed by potential customers, but the production process that led to that advertisement is not observed. As

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we will show in Section 5, an agency would like to publicly post a price to manipulate the receiver's beliefs but would then want to secretly change that price.

3   Demand for Rhetorical Services

This section derives the demand curve for rhetorical services. In the process of doing so, we highlight how the service affects receiver welfare and how receiver beliefs affect demand. Both this and subsequent sections mostly focus on two contrasting types of rhetorical services in order to clearly highlight the equilibrium implications of these services.

If the receiver believes that the sender purchases the rhetorical service with probability $Q \in [0, 1]$, then after observing s, he believes that $\omega = 1$ with probability $\mu^*(s|Q)$, which is a strictly increasing function of the likelihood ratio

$$l^*(s|Q) \equiv \frac{\gamma}{1 - \gamma} \frac{Qg_1(s) + (1 - Q)f_1(s)}{Qg_0(s) + (1 - Q)f_0(s)}. \tag{1}$$

Since $u_R(\cdot)$ exhibits strictly increasing differences, the receiver's optimal decision is a strictly increasing function of this likelihood ratio. We can therefore write the sender's expected payoff as a function of this likelihood ratio, $u_S^*(l, t)$, where $u_S^*(\cdot)$ is strictly increasing in l and exhibits strictly increasing differences.

Since higher types are willing to pay more to induce higher posteriors, the sender purchases rhetorical services whenever her type exceeds a threshold.

Lemma 1 *In any equilibrium, there exists a $t^* \in \mathbb{R}$ such that $x = 1$ if and only if $t \geqslant t^*$, and the receiver's equilibrium belief after observing signal s has likelihood ratio $l^*(s|1 - t^*)$.*

The proofs for all results may be found in Appendix A. We can define the expression

$$\Delta(t|Q) \equiv \int_s u_S^*(l^*(s|Q), t) [\gamma(g_1(s) - f_1(s)) + (1 - \gamma)(g_0(s) - f_0(s))] \, ds \tag{2}$$

as the **willingness-to-pay** of a sender of type t if the receiver believes that the service is purchased with probability Q. Note that receiver beliefs affect sender willingness-to-pay only through Q.

Most of our analysis focuses on two contrasting types of rhetorical services. We say that the rhetorical service **clarifies** if $f_0(\cdot) = g_0(\cdot)$, so that it affects the signal distribution only if $\omega = 1$. The service **embellishes** if $f_1(\cdot) = g_1(\cdot)$ so that it changes the signal distribution only if $\omega = 0$. The sender values both embellishing and clarifying services because they lead to higher signal realizations, but how exactly they do so has important implications for demand, pricing, and welfare.

Our next result demonstrates how receiver beliefs influence sender willingness-to-pay. We show that demand for a clarifying rhetorical service is increasing in the receiver's beliefs that the sender purchases that service, while demand for embellishing services is decreasing in the receiver's beliefs about the probability of purchase.

Proposition 1 *Fixing Q, the sender's willingness-to-pay is strictly increasing in t. Fixing t, her willingness-to-pay is strictly increasing in Q if the service clarifies and strictly decreasing in Q if the service embellishes.*

The sender's willingness-to-pay is strictly increasing in t because $u_S^*(l, t)$ exhibits strictly increasing differences. The receiver's beliefs, Q, affect the posterior she assigns to each signal and therefore influence sender willingness-to-pay through $l^*(s|Q)$. For a clarifying service, an increase in Q leads the receiver to assign higher posteriors to exactly those signals that the service makes more likely, which increases the value of those signals to the sender. Receiver beliefs have the opposite effect for embellishing services: an increase in Q leads to *lower* posteriors following those signals that the service makes more likely and so decreases willingness-to-pay.

Proposition 1 highlights the role of the receiver's beliefs on market outcomes, which differentiates rhetorical services from most other products. The next set of results characterize

the implications of this observation for demand. Noting that higher types are willing to pay more for the service, we define the **equilibrium demand curve** as

$$\Delta^*(Q) \equiv \Delta\left(1 - Q|Q\right).$$

That is, $\Delta^*(Q)$ equals the sender's willingness-to-pay if her type is $1 - Q$ and the receiver believes that the service is purchased with probability Q. Given a price p, the purchase probabilities consistent with a continuation equilibrium consist of those Q that satisfy $\Delta^*(Q) = p$, as well as $Q = 0$ if $\Delta^*(0) < p$ and $Q = 1$ if $\Delta^*(1) > p$. Denote this set of purchase probabilities by $\mathcal{Q}(p)$.

We seek conditions under which the equilibrium demand curve is either upward- or downward-sloping. We can decompose the slope of this curve into two parts,

$$\frac{d\Delta^*(Q)}{dQ}\big|_{Q=Q^*} \equiv -\frac{\partial\Delta(t|Q^*)}{\partial t}\big|_{t=1-Q^*} + \frac{\partial\Delta(1-Q^*|Q)}{\partial Q}\big|_{Q=Q^*}. \tag{3}$$

The first term in (3) is the **direct effect** of increasing the probability of sale for fixed receiver beliefs, which is always negative because lower types are willing to pay strictly less for the service. The second term captures the **strategic effect** as receiver beliefs adjust in equilibrium. Proposition 1 implies that the strategic effect is negative if the service embellishes but is positive if the service clarifies. Consequently, equilibrium demand is always downward-sloping for embellishing services but may be either downward- or upward-sloping for services that clarify.

Proposition 2 *If the service embellishes, then equilibrium demand is strictly decreasing in Q and $\mathcal{Q}(p)$ has at most one element for any p. If the service clarifies, then equilibrium demand is strictly increasing in Q if and only if the strategic effect is larger than the direct effect. If equilibrium demand is ever strictly increasing in Q, then there exists some p for which $\mathcal{Q}(p)$ has more than one element.*

Proposition 2 follows immediately from (3) and the equilibrium spillovers identified in Proposition 1. Any price that intersects both downward- and upward-sloping parts of the equilibrium demand curve is associated with multiple elements of $\mathcal{Q}(p)$ and so multiple possible purchase probabilities. For a clarifying service, demand is upward-sloping wherever the direct effect is small relative to the strategic effect. This condition is likely to hold if different sender types have similar preferences, in the sense that $\partial u_S^*/\partial t$ is close to 0.[5]

The final result in this section considers how the rhetorical service affects the receiver's utility. In equilibrium, the **receiver's welfare** can be expressed as a function of the probability of purchase Q,

$$u_R^*(Q) \equiv E_{s,\omega}\left[u_R\left(d^*\left(l^*(s|Q)\right),\omega\right)|Q\right].$$

The next result shows that, if the rhetorical service clarifies, then a higher purchase probability leads to a more informative equilibrium signal distribution and hence higher receiver welfare. The opposite result holds if the service embellishes, in which case its purchase deteriorates the information conveyed in equilibrium and so harms the receiver.

Proposition 3 *Receiver welfare is strictly increasing in Q if the service clarifies and strictly decreasing in Q if it embellishes.*

Since the receiver's decision problem is monotone, his preferences over information structures can be ranked using the Lehmann ordering (Lehmann (1988); Athey and Levin (2018)). Proposition 3 is a corollary of Lehmann (1988), Theorem 5.1. As its name suggests, the sender values a clarifying rhetorical service precisely because it "disentangles" signals in the high and low states, resulting in a more informative mapping from state to signal distribution. In contrast, an embellishing service pools low- and high-state signals and leads to a less informative signal.

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[5]We can construct examples to show that upward-sloping equilibrium demand can indeed arise. In particular, if $\partial u_S^*/\partial t = 0$, then equilibrium demand is everywhere upward-sloping.

4 Supply of Rhetorical Services

We have shown that market outcomes impact both the sender's (Proposition 1) and the receiver's (Proposition 3) expected payoffs. In this section, we show that agencies do not internalize either of these spillovers when they set prices, and we explore the implications of this fact for the supply of rhetorical services. Proofs for this section are in Appendix A.

The Monopolist's Problem: We begin by considering the monopolist agency's profit maximization problem. A monopolist agency is free to price along the sender's demand curve for fixed receiver beliefs, but it cannot influence those beliefs because the receiver does not observe prices. Let $t^*(p|Q)$ be the unique **marginal sender** given price p and probability of sale Q, defined by $\Delta\left(t^*(p|Q)|Q\right) = p$. Let $t_p^*(p|Q)$ denote the partial derivative of $t^*(p|Q)$ with respect to p. The results in this section will require that any equilibrium entails an interior probability of purchase. Formally, we say that the game has **interior market outcomes** if $\Delta(0|Q) > c \geq \Delta(1|Q)$ for any $Q \in [0,1]$.

Our next result characterizes the profit-maximizing price in terms of this type.

Proposition 4 *Suppose the game has interior market outcomes. In any equilibrium,*

$$p^* = c + \frac{Q^*}{t_p^*(p^*|Q^*)}, \tag{4}$$

where $Q^* = 1 - t^*(p^*|Q^*)$.

The expression (4) is a standard monopoly markup formula with the additional condition that the receiver's beliefs about the probability of sale are correct in equilibrium. This additional condition highlights the first of our equilibrium spillovers: the agency chooses its price for fixed beliefs, but in equilibrium, beliefs are correct and so the equilibrium price is reflected in the demand curve. Together with Proposition 3, Proposition 4 also highlights the second equilibrium spillover: the agency's price affects the welfare of a third party, the

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receiver, who has no direct say in the market transaction. These two spillovers suggest that profit-maximizing behavior may lead to dysfunctional market outcomes, a point that we explore in the rest of this section.

Competition Among Agencies: Our next result considers the role of competition in mitigating or exacerbating these spillovers. Instead of the **monopolist market** analyzed above, consider a **Bertrand market** in which it is commonly known that two or more agencies with identical costs simultaneously set prices. The sender can choose to purchase services from at most one of these agencies, where purchasing a service changes the signal distribution from $F_\omega(\cdot)$ to $G_\omega(\cdot)$. Competition among agencies drives price to marginal cost and increases adoption, but Proposition 3 implies that increased adoption does not necessarily benefit the receiver. Even the sender might be hurt by competition, as the benefits from lower prices might be outweighed by the impact of competition on purchase decisions and hence receiver beliefs.

Proposition 5 *Assume the game has interior market outcomes. For any equilibrium of the monopolist market, there exists an equilibrium of the Bertrand market in which receiver welfare is higher if the service clarifies or lower if the service embellishes. Moreover, examples exist in which every equilibrium outcome of the monopolist market Pareto dominates every equilibrium outcome of the Bertrand market.*

The effect of competition on the receiver's welfare follows immediately from Proposition 3 and the fact that lower prices increase the probability of purchase in equilibrium. The second part of Proposition 5 says that increasing competition can decrease *all* players' payoffs. Though narrow in scope, this striking result illustrates how equilibrium spillovers complicate the relationship between competition and welfare. We prove it by considering a setting in which $c = 0$, so that the sender purchases the service with probability 1 under Bertrand competition. The agency clearly does not benefit from competition, and the receiver does not benefit if the service embellishes. If $F_1(\cdot) \approx G_1(\cdot) \approx G_0(\cdot)$, then communication is

approximately uninformative if the sender purchases the product with probability 1. If the sender earns close to zero unless the receiver's posterior is substantially larger than γ, then competition leads to nearly uninformative equilibrium signals and hence a very low expected payoff for the sender. With a monopolist agency, in contrast, the sender purchases the product with probability strictly less than 1. Therefore, communication remains informative and so the sender earns a strictly positive payoff regardless of her purchase decision.

Monopoly with Public Prices: Our next result shows how making prices public leads the agency to internalize the spillover from receiver beliefs to sender demand. To make this point, we consider a monopolist market with **public prices,** which is identical to the model in Section 2 except that p is publicly observed. The agency's price therefore affects receiver beliefs, with the consequence that it faces the *equilibrium* demand curve rather than a demand curve with fixed beliefs. Along the upward-sloping regions of the equilibrium demand curve, we select the **agency-optimal** continuation equilibrium, defined as the continuation equilibrium with the *largest* equilibrium probability of purchase. Given this refinement, the effective demand curve is given by $Q^*(p) = \max\{Q \in \mathcal{Q}(p)\}$ and so is again downward-sloping.[6]

Proposition 6 shows that the agency increases the price of embellishing services in order to decrease the probability of sale and so increase willingness-to-pay. It decreases the price of clarifying services for a similar reason.

Proposition 6 *Assume the game has interior market outcomes. Let p_{PVT} be any equilibrium price in the monopoly market. Let p_{PUB} be any agency-optimal equilibrium price in the market with public prices. If the service embellishes, $p_{PUB} > p_{PVT}$. If the service clarifies, $\mathcal{Q}(p_{PVT})$ is a singleton on a neighborhood around p_{PVT}, and $\frac{d\Delta^*(Q)}{dQ}|_{Q=Q^*(p_{PVT})} \neq 0$, then $p_{PUB} < p_{PVT}$.*

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[6]The market with public prices is closely related to a market for network goods (as in Akerlof et al. (2018) and others) because beliefs respond to prices. In the language of Akerlof et al. (2018), this equilibrium refinement selects the "in" demand curve.

The monopolist's price balances the value of a higher purchase probability against the loss from a lower price charged to inframarginal sender types. Proposition 2 implies that equilibrium demand for embellishing services "falls faster" than $\Delta(t|Q)$. Making prices public means that the agency has to reduce its price by *more* to achieve a given increase in purchase probability, which leads to higher equilibrium prices. An analogous argument proves the result for clarifying services, with the important complication that $Q^*(\cdot)$ might not be differentiable. However, this complication does not arise if $\mathcal{Q}(p_{PVT})$ is a singleton and $\frac{d\Delta^*(Q)}{dQ}|_{Q=Q^*(p_{PVT})} \neq 0$.[7]

Design of Rhetorical Services: Our final result in this section considers how the spillovers identified above shape the profit-maximizing design of rhetorical services. To that end, we consider a **product design game,** which is identical to the monopolist market except that the agency can costlessly choose $G_\omega(\cdot)$ to be *any* distribution at the same time it chooses p. The sender observes the chosen $G_\omega(\cdot)$ but the receiver does not. While extreme, the assumption that the agency can design any rhetorical service at no cost starkly illustrates how the agency's optimal choices can lead to self-defeating equilibrium outcomes.[8]

For fixed receiver beliefs, the agency optimally chooses a rhetorical service with support on only those signals that induce the highest posteriors. But then the rhetorical service essentially makes the signal uninformative about the underlying state, which limits the price that the agency can charge for it in equilibrium. Consequently, the possibility of product design leads to less informative communication, to the detriment of the receiver and potentially of the agency as well. Our next result demonstrates this intuition for the case of homogeneous senders and $c = 0$.

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[7]If the sender's purchase decision, rather than the price, was public, rhetorical services would play a different role. In that case, these services would change the continuation game from one in which it is common knowledge that $s \sim F_\omega(\cdot)$ to one in which it is common knowledge that $s \sim G_\omega(\cdot)$. The value of such a service would depend on how the sender values the resulting posterior distribution, given that the receiver knows the true mapping from state to signal.

[8]Unlike the baseline model, the agency might benefit from offering a menu of different rhetorical services in this setting. We restrict the agency from doing so here in order to better compare this setting to the baseline model.

Proposition 7 *Suppose that $c = 0$ and $\partial u_S / \partial t = 0$ for all (d, t). In any equilibrium, the receiver's posterior belief always equals γ, and at least one of the following two conditions holds: (i) $p = 0$, or (ii) the sender purchases the service with probability 1. There exists an equilibrium in which both of these conditions hold.*

The first step in the proof of Proposition 7 is to show that either $p = 0$, or the sender purchases the service with probability 1. If not, then the agency can profitably reduce its price to induce a discretely higher purchase probability because the sender's willingness-to-pay is both commonly known (because $\partial u_S / \partial t = 0$) and strictly positive. Next, we show that equilibrium communication must be uninformative, as the agency maximizes its profit by choosing a $G_\omega(\cdot)$ that puts weight only on those signals that induce the highest possible posterior belief. If that belief is strictly larger than the prior, then the sender is willing to pay a strictly positive price, which by the previous argument implies that the sender purchases with probability 1. But then the equilibrium signal is completely uninformative because the rhetorical service induces the same posterior belief regardless of the state. If the chosen $G_\omega(\cdot)$ has full support, then *all* signals induce the same belief and so $p = 0$.

Proposition 7 is related to Lizzeri (1999)'s study of optimal certification in the presence of allocative inefficiencies (Theorem 5 in that paper). However, the purchase of a service cannot directly signal anything in our setting because our receiver never observes the product design or the sender's purchase decision. Consequently, while Lizzeri (1999) finds that the certifying agency reveals some information and earns a strictly positive profit, our agency reveals no information and might earn zero profit.[9]

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[9] Other equilibria potentially exist in our setting, including some in which the agency earns strictly positive profit. However, the beliefs that sustain those equilibria are delicate. In particular, these equilibria require that $G_\omega(\cdot)$ has support on a subset of the space of signal, the receiver believes that any signal not in this set perfectly reveals $\omega = 0$, and the sender purchases with probability 1. These beliefs are sustained by the following off-path conjecture: after observing any off-path signal, the receiver believes that the agency deviated by designing a perfectly informative $G_\omega(\cdot)$ which assigned the observed signal to $\omega = 0$.

5 Regulation of Markets for Rhetorical Services

A key takeaway from our analysis so far is that markets for rhetorical services do not necessarily maximize the payoffs of the receiver or even of the market participants. This section considers how well-designed regulation can mitigate these inefficiencies, focusing on interventions that render false arguments harder to make. Education campaigns and truth-in-advertising laws increase the difficulty of making persuasive but false arguments, the former by helping receivers better detect incorrect arguments and the latter by penalizing the sender for making them. In politics, analogous regulations might combat "astroturfing" tactics that falsely claim popular grassroots support for an industry's preferred policy, for instance by requiring transparency about how such tactics are funded.[10] Proofs for this section are again in Appendix A.

We model these regulations as shifting the distribution of signals following $\omega = 0$ downward. Formally, we parameterize a family of distributions $F_0^r(\cdot)$ and $G_0^r(\cdot)$ by the parameter $r \in [0, 1]$. A higher r corresponds to a "tighter" regulation in the sense that $F_0^r(\cdot)$ and $G_0^r(\cdot)$ are decreasing in r in the sense of first-order stochastic dominance. In contrast, $F_1(\cdot)$ and $G_1(\cdot)$ are unaffected by the regulation. We assume that both $F_0^r(\cdot)$ and $G_0^r(\cdot)$ are twice continuously differentiable in r. Given these assumptions, tighter regulation have a positive direct effect on the receiver's welfare, since increasing r leads to a Lehmann more informative distribution for fixed Q. Our analysis considers how regulation affects market outcomes. We show that regulation increases the sender's returns from higher signal realizations, which changes the sender's benefit from purchasing the rhetorical service.

Some types of regulations depress demand for some types of rhetorical services. For example, if the regulation perfectly reveals $\omega = 0$ with probability r, then increasing r decreases

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[10] A recent investigation by the television show *Last Week Tonight* suggests that, as with the "merchants of doubt" from Oreskes and Conway (2011), a small group of publicity firms, such as *Crowds on Demand*, organize many astroturfing campaigns (Oliver (2018)). On its website, *Crowds on Demand* boasts that it "was hired by multiple large non-union firms to push back against new regulations in a deeply labor-friendly state....we created two organizations with associated websites....Within two months, the proposed regulations were off the table."

demand for embellishing rhetorical services.[11] Our next result shows that regulation can also *increase* demand in the market by increasing the returns to inducing high posteriors. We impose several simplifications to highlight this implication. For each $r \in [0, 1]$, define the signal's likelihood ratio as

$$l^r(s|Q) = \frac{\gamma}{1 - \gamma} \frac{Qg_1(s) + (1 - Q)f_1(s)}{Qg_0^r(s) + (1 - Q)f_0^r(s)},$$

and define willingness-to-pay, $\Delta^r(t|Q)$, as in (2).

Proposition 8 *Let Assumptions 1 and 2 hold for each $r \in [0, 1]$. Suppose that $u_S^*(l, t) = lt$ and that $l^r(s|Q)$ satisfies strictly increasing differences in r and s for any Q. Then $\Delta^r(t|Q)$ is strictly increasing in r for any t and Q if either (i) the service clarifies, or (ii) $\frac{\partial g_0^r}{\partial r}$ strictly single-crosses $\frac{\partial f_0^r}{\partial r}$ from below.*

The proof of Proposition 8 highlights two reasons why regulation changes demand for rhetorical services. First, tighter regulation increases the sender's return from a higher signal realization, since $l^r(s|Q)$ exhibits strictly increasing differences in r and s and $u_S^*(l, t)$ is linear in l. Second, regulation affects the extent to which purchasing the rhetorical service generates higher signal realizations relative to not purchasing it. If the rhetorical service clarifies, then it does not affect the signal distribution in the low state, in which case only the former effect is relevant and so demand is increasing in the regulation. If the service does not clarify, then assuming that $\frac{\partial g_0^r}{\partial r}$ single-crosses $\frac{\partial f_0^r}{\partial r}$ from below essentially means that the regulation affects $f_0^r(\cdot)$ "more" than $g_0^r(\cdot)$. In that case, tighter regulation increases demand because it both (i) increases the returns to a higher signal realization and (ii) increases the amount by which purchasing the rhetorical services increases the signal.

Our analysis assumes that the sender's expected payoff is linear in l, so it should be interpreted as illustrating a possible, rather than an inevitable, consequence of regulation.

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[11]To model this type of regulation, assume that $F_0^r(\cdot) = (1 - r)F_0^0(\cdot) + rF^D(\cdot)$ and $G_0^r(\cdot) = (1 - r)G_0^0(\cdot) + rF^D(\cdot)$, where $F^D(\cdot)$ is the degenerate distribution that assigns probability 1 to $s = 0$.

Propositions 3 and 8 nevertheless suggest that ostensibly welfare-improving regulation might change market outcomes in ways that either augment or undermine those welfare benefits. For instance, tighter regulation might spur the sender to purchase rhetorical services with higher probability.[12] If those services clarify, then a higher purchase probability compounds the positive effects of the regulation and so a relatively small regulation could generate large improvements in receiver welfare. On the other hand, if the sender responds by purchasing more embellishing services, then the regulation's direct effect on receiver welfare would be muted by a countervailing market response.

6   Rhetorical Ability in a Sender-Receiver Game

This section proposes a communication game as a way to better understand the signal distributions $F_\omega(\cdot)$ and $G_\omega(\cdot)$ in the market for rhetorical services. Building on a standard sender-receiver model (Sobel (2013)), we propose a measure of an argument's **strength**, and we give the sender private information about her **rhetorical ability**, which constrains the strength of her arguments. We argue that strong arguments are persuasive because they are hard to make if they are not true, but they are not completely so because even strong arguments can be co-opted by rhetorically gifted liars. In a receiver-optimal equilibrium, the distribution over argument strengths mimics the signal distribution in the market for rhetorical services, providing a microfoundation for $F_\omega(\cdot)$ and $G_\omega(\cdot)$.

6.1   A Model of Rhetorical Ability

Consider a sender who wants to persuade a receiver that a binary state of the world is high. The sender is informed about the state and can make an **argument** $a = (m, s)$ to the receiver. An argument consists of a **message** $m \in \{0, 1\}$ and a **strength** $s \in \mathbb{R}$. The sender can send any message in any state, but the maximum strength of the resulting argument

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[12]Under the conditions of Proposition 8, the probability of purchase is increasing in demand in the Bertrand market, but not necessarily in the monopolist market.

depends on both the state $\omega \in \{0, 1\}$ and the sender's privately observed **rhetorical ability,** $\theta = (\theta_T, \theta_L)$. We assume that $s \le \theta_T$ if $m = \omega$ and $s \le \theta_L$ otherwise, and we interpret θ_T and θ_L as the sender's truth-telling and lying ability, respectively. That is, the sender's rhetorical ability consists of two numbers, the first of which is the strongest truthful argument she can make and the second of which is her strongest lying argument. We denote the resulting set of feasible arguments by

$$\mathcal{A}(\theta, \omega) = \mathcal{A}^T(\theta_T, \omega) \cup \mathcal{A}^L(\theta_L),$$

where $\mathcal{A}^T(\theta_T, \omega) = \{(\omega, s) | s \le \theta_T\}$ is the set of truthful arguments the sender can make, and $\mathcal{A}^L(\theta_L) = \{(m, s) | m \in \{0, 1\}, s \le \theta_L\}$ is the set of arguments that she can make regardless of the state.

Our analysis follows from two assumptions. First, the sender can make stronger truthful arguments than false ones, so the receiver might learn about ω by observing an argument's strength in equilibrium. Second, rhetorically able senders might be able to make false arguments that are stronger than the true arguments of less able senders. Consequently, equilibrium communication is not perfectly informative, since a sender who is talented at lying can pool with one who is telling the truth.

Formally, consider a **communication game** between the sender and receiver with the following timing.

1. The sender privately learns her **rhetorical ability** $\theta \equiv (\theta_T, \theta_L) \in \Theta \subseteq \mathbb{R}^2$, with $\theta \sim F(\cdot)$, and the **state** $\omega \in \{0, 1\}$, with $\Pr\{\omega = 1\} = \gamma$.

2. The sender makes an **argument** $a \equiv (m, s) \in \mathcal{A}(\theta, \omega)$.

3. The receiver observes a and makes a **decision** $d \in \mathbb{R}$.

Payoffs are $v_S(d)$ and $v_R(\omega, d)$ for sender and receiver, respectively. As in Section 2, we assume $v_S(\cdot)$ is strictly increasing and $v_R(\cdot)$ exhibits strictly increasing differences. Without loss, we assume $\theta_T \ge \theta_L$ for all θ in the support of $F(\cdot)$. Let $F_1(\cdot)$ and $F_0(\cdot)$, with densities

$f_1(\cdot)$ and $f_0(\cdot)$, be the marginal distributions over θ_T and θ_L, respectively. It is not a coincidence that we use "s" for "signal" in Section 2 and "strength" here, as an equilibrium distribution over strengths in this communication game coincides with the signal distributions in the market for rhetorical services.

Discussion of the Communication Game

This model nests both cheap talk and verifiable disclosure as limiting cases. Cheap talk (Crawford and Sobel (1982); Lipnowski and Ravid (2017)) obtains if $\theta_T = \theta_L$ for all θ in the support of $F(\cdot)$, so that it is common knowledge that the set of feasible arguments is independent of the state. Verifiable disclosure (Milgrom (1981); Milgrom and Roberts (1986b)) obtains if, for example, $\theta_T = 1 > \theta_L$ for all θ in the support of $F(\cdot)$, so that the argument $(1, 1)$ can be sent if and only if $\omega = 1$.[13]

More generally, our model captures a "middle ground" between verifiable disclosure, in which an argument incontrovertibly establishes the truth of a claim, and cheap talk, in which messages have no supporting argument. We believe it is therefore well-suited to study communication that depends on the judgment and skill of the sender and the beliefs of the receiver, as in much of law, politics, and advertising (Aldisert et al. (2007)). One interpretation of θ is that it equals the *number* of arguments available to the sender. In this interpretation, the sender can devise θ_L "clever" arguments in favor of $\omega = 1$ regardless of the truth. If the state is in fact $\omega = 1$, then the sender can also devise $\theta_T - \theta_L$ "logical" arguments in favor of $\omega = 1$. The receiver observes the number of arguments that the sender chooses to make, but he cannot tell whether those arguments are logical or clever.

Figure 1 gives illustrative examples of logical and clever arguments in different contexts. Our categorization is potentially controversial, as the reader might believe that some logical arguments are instead clever, or vice versa. This ambiguity actually strengthens our central

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[13]Relative to many of the papers that study cheap talk or verifiable disclosure, note that we make simpler assumptions about both the state of the world (binary) and the sender's preferences (monotonic in the receiver's posterior belief).

	Logical	Clever
Advertising	"This car has the best highway gas mileage of cars in its class."	"It's toasted!" [Lucky Strike cigarettes]
	"Independent clinical studies show…"	"9 out of 10 dentists can't be wrong!" [for toothpaste]
		"No matter what your doubts. Remember that my method is unknown to you, while I spent a lifetime on it." [Dr. Shoops Restorative]
Law	"The Judicial Department determines the law; the Supreme Court is the Judicial Department; therefore, the Supreme Court's duty is to say what the law is." [Marbury v. Madison, adapted from Aldisert, Clowney, and Peterson (2007)]	"If it [the glove] doesn't fit, you must acquit." [OJ Simpson trial]
	"But justice in this case has nothing to do with some of the arguments put forth by the defendants or their counsel….We are not trying them for possession of obnoxious ideas….It is not their thoughts, it is their overt acts which we charge to be crimes." [Robert Jackson, at the Nuremberg trial in 1946, as recorded in Lief, Caldwell, and Bycel (1999)]	"I am not on trial for having sought to bribe a man named Lockwood….I am on trial because I have been a lover of the poor, a friend of the oppressed, because I have stood by labor for all these years, and have brought down upon my head the wrath of the criminal interests in this country." [Clarence Darrow, in his closing argument in his own defense against a bribery accusation, as recorded in Lief, Caldwell, and Bycel (1999)]
Politics	"I am not included within the pale of this glorious anniversary! Your high independence only reveals the immeasurable distance between us….the rich inheritance of justice, libery, prosperity, and independence bequeathed by your fathers is shared by you, not me." [Frederick Douglass, "The Hypocrisy of American Slavery"]	"I have here in my hand a list of 205 [State Department employees] that were known to the Secretary of State as being members of the Communist Party and who nevertheless are still working and shaping the policy of the State department." [Joseph McCarthy, 1950. McCarthy's claimed number fluctuated wildly in the following weeks, and he never produced solid evidence to back it up.]

Figure 1: Examples of logical and clever arguments.

claim, which is that it is difficult for a listener to cleanly separate logical and clever arguments, so that a clever argument might persuade even if it is not actually evidence of the underlying state.[14]

This model is especially related to Kartik (2009) and Frankel and Kartik (2017), both of which study settings in which lying is costly and so also nest cheap talk and verifiable communication.[15] As in Dziuda (2011), Blume and Board (2013), and Hagenbach et al. (2014), our communication model assumes that the sender has private information about the set of feasible arguments. Like Dewan and Myatt (2008), we develop a notion of what it means to be a "better" or "worse" communicator.

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[14]Note that clever arguments are made to support *both* true and false messages in our framework, so categorizing an argument as "clever" does not imply that the underlying claim is false.

[15]Frankel and Kartik (2017) assume that the cost of lying is private information, which can be viewed as an intermediate case between cheap talk and our own assumption that the sender has private information about which messages are feasible.

6.2 Equilibrium Persuasion

This section shows how the argument distribution in the receiver-optimal equilibrium of the communication game replicates the signal distribution from Section 2. Proofs may again be found in Appendix A.

Consider the following strategy: regardless of the true state, the sender makes the strongest argument that the state is high, so $a = (1, \theta_T)$ if $\omega = 1$ and $a = (1, \theta_L)$ if $\omega = 0$. If the receiver believes that the sender uses this strategy, then her posterior belief that the state is high equals $\mu^*(s|0)$ corresponding to likelihood ratio (1) with $Q = 0$, which is increasing in s by Assumption 2. If the receiver believes $\omega = 0$ whenever he observes $m = 0$, then the sender has no profitable deviation from this strategy, which is therefore an equilibrium.

Our next result proves that this equilibrium maximizes the receiver's expected payoff among all equilibria.

Proposition 9 *Let Assumptions 1 and 2 hold in the communication game. Any equilibrium in which the receiver's on-path posterior belief equals $\mu^*(\theta_T|0)$ if $\omega = 1$ and $\mu^*(\theta_L|0)$ if $\omega = 0$ maximizes the receiver's expected payoff among all equilibria.*

To prove Proposition 9, we show that an equilibrium with the desired features is more informative than any other PBE in the Blackwell sense. This argument is somewhat involved because the sender's equilibrium strategy can depend on both state and type in potentially complicated ways. We show that the receiver must have the same posterior following $\omega = 1$ and $\theta_T = s$ or $\omega = 0$ and $\theta_L = s$ in any equilibrium, a result that relies on the fact that senders can always weaken their arguments and can always make stronger truthful relative to lying arguments. Therefore, the receiver cannot distinguish $\omega = 1$ and $\theta_T = s$ from $\omega = 0$ and $\theta_L = s$ in *any* equilibrium. The equilibria described by Proposition 9 pool *only* these events together, so they must convey more information than any other PBE.

In the receiver-optimal equilibrium described above, all on-path arguments are of the form $a = (1, s)$, where $s = \theta_T \sim F_1(\cdot)$ if $\omega = 1$ and $s = \theta_L \sim F_0(\cdot)$ if $\omega = 0$. That is,

the equilibrium distribution of strengths exactly replicates the signal distribution from our model of the market for rhetorical services. The agency can therefore be interpreted as improving the sender's rhetorical ability so that $\theta_T \sim G_1(\cdot)$ and $\theta_L \sim G_0(\cdot)$, where clarifying and embellishing services allow the sender to make stronger logical or clever arguments, respectively. Truth-in-advertising laws and public education campaigns both decrease θ_L by allowing the receiver to better identify whether an argument is true or not.

In Appendix B, we discuss other equilibria and expand our analysis to settings in which Assumption 2 does not hold. The main takeaway from Proposition 9 is that we can micro-found the market for rhetorical services in a sender-receiver model. The key twist of this model is a notion of rhetorical ability, which then allows us to interpret rhetorical services as improving this ability and hence the strength of the sender's equilibrium argument.

7 Conclusion

We view our analysis as a first step towards the ultimate goal of understanding markets for rhetorical services. To that end, we briefly discuss three enrichments of the model.

First, in the application to advertising markets, the sender is a firm and so her incentives to purchase rhetorical services presumably depend on her industry's market structure. One could extend the model to analyze (i) competition among senders, (ii) whether competing senders treat rhetorical services as strategic complements or strategic substitutes, (iii) when and how they might use rhetorical services to deter entry, and (iv) how access to those services shapes the intensity of price competition.

Second, the example of Dr. Shoop's Restorative suggests that receivers learn from their past experiences with rhetorical techniques. The advent of a new communication technology—be it mass mailing campaigns, radio, television, or the Internet—is typically accompanied by uncertainty about the extent to which that technology can be manipulated. The recent concerns over "fake news" and viral propaganda on social media illustrate how

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strategic senders might take advantage of credulous receivers who do not yet understand the types of lies that can be made on a platform. As a medium matures, however, audiences potentially learn which types of arguments are actually trustworthy and which are susceptible to manipulation. Analyzing these dynamics would require a model in which the receiver is initially uncertain about $G_\omega(\cdot)$ but learns about it as he interacts with senders.

Finally, the communication game in Section 6 could be used to analyze rhetoric and persuasion in organizations, capital markets, or politics. As a new product diffuses across society, how does an early adopter's rhetorical ability affect the extent of its eventual success? How does the presence of a charismatic founder or early investor in a start-up affect follow-on investment and the eventual success of that firm? How do employees wield their rhetorical talents to manipulate firm decisions, and how should management structure its incentives and hierarchy to take advantage of those talents? This communication game provides a building block to address these and related questions.

Rhetorical services are fundamentally different from other products because their value depends on the beliefs of their intended audience. Since purchases in these markets make up a substantial fraction of all economic transactions, and since those purchases generate spillovers that influence other economic, political, and social outcomes, it is important to understand how these markets operate. Our framework provides a simple but systematic way to model markets for rhetorical services, in the hopes of ultimately understanding how demand and supply in these markets shape society at large.

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A   Omitted Proofs

Given purchase probability Q, the receiver's posterior after observing signal s equals

$$\mu^*(s|Q) \equiv \frac{Qg_1(s) + (1-Q)f_1(s)}{Qg_1(s) + (1-Q)f_1(s) + \frac{1-\gamma}{\gamma}\left(Qg_0(s) + (1-Q)f_0(s)\right)}. \tag{5}$$

A.1   Proof of Lemma 1

To prove the result, we first show that the sender's willingness-to-pay for the service as defined in (2) is increasing in t. Taking a derivative with respect to t yields

$$\frac{\partial\Delta}{\partial t}(t|Q) = \int_s \frac{\partial u_S^*}{\partial t}(l^*(s|Q), t)\left[\gamma(g_1(s) - f_1(s)) + (1-\gamma)(g_0(s) - f_0(s))\right]ds.$$

Now, $\frac{\partial^2 u_S^*}{\partial s \partial t} \equiv \frac{\partial^2 u_S^*}{\partial l \partial t}\frac{\partial l}{\partial s} > 0$ because $u_S^*(l, t)$ exhibits strictly increasing differences and $\frac{\partial l}{\partial s} > 0$. Therefore, $\frac{\partial u_S^*}{\partial t}(l^*(s|Q), t)$ is strictly increasing in s. By Assumption 2, $g_1(s) - f_1(s)$ and $g_0(s) - f_0(s)$ single-cross 0 from below, with at least one strictly so, and $\int_s(g_1(s) - f_1(s))ds = \int_s(g_0(s) - f_0(s))ds = 0$. Therefore, $\frac{\partial\Delta}{\partial t}(t|Q) > 0$ by the strict version of Beesack's Inequality.[16] We conclude that if some type t purchases the service in equilibrium, then so does every type $t' > t$.

Fix an equilibrium, and suppose that the sender purchases the service if and only if $t \geqslant t^*$. The receiver's believes that the sender purchases with probability $Q = 1 - t^*$, which means that signal s induces posterior belief $\mu^*(s|1 - t^*)$, which has likelihood ratio $l^*(s|1 - t^*)$. ∎

A.2   Proof of Proposition 1

The proof of Lemma 1 implies that $\Delta(t|Q)$ is strictly increasing in t. For any t, $u_S^*(l^*(s|Q), t)$ is strictly increasing in s, so the strict version of Beesack's Inequality implies that (2) is strictly positive.

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[16]The relevant version of Beesack's Inequality states that if a function $\gamma(\cdot)$ single-crosses 0 from below and satisfies $\int \gamma(x)dx = 0$, then for any increasing function $\lambda(\cdot)$, $\int \gamma(x)\lambda(x)dx \geqslant 0$, and strictly so if $\lambda(\cdot)$ is strictly increasing and $\gamma(\cdot)$ is not everywhere 0. See Beesack (1957).

Suppose the service clarifies, so $g_0(s) = f_0(s)$. Then

$$\frac{\partial \Delta}{\partial Q} = \gamma \int_s \frac{\partial u_S^*}{\partial l}(l^*(s|Q), t)\frac{\partial l^*}{\partial Q}(s|Q)\left(g_1(s) - f_1(s)\right)ds.$$

In this case, $\frac{\partial l^*}{\partial Q}(s|Q) = \frac{\gamma}{1-\gamma}\frac{g_1(s)-f_1(s)}{f_0(s)}$, so

$$\frac{\partial \Delta}{\partial Q} = \frac{\gamma^2}{1-\gamma}\int_s \frac{\partial u_S^*}{\partial l}(l^*(s|Q), t)\frac{\left(g_1(s) - f_1(s)\right)^2}{f_0(s)}ds.$$

But $\frac{\partial u_S^*}{\partial l} > 0$, so $\frac{\partial \Delta}{\partial Q} > 0$ as desired.

Suppose the service embellishes so that $g_1(s) = f_1(s)$. Then $\frac{\partial l^*}{\partial Q}(s|Q) = -\frac{\gamma}{1-\gamma}\frac{f_1(s)(g_0(s)-f_0(s))}{(Qg_0(s)+(1-Q)f_0(s))^2}$ and so

$$\frac{\partial \Delta}{\partial Q} = -\gamma \int_s \frac{\partial u_S^*}{\partial l}(l^*(s|Q), t)\frac{f_1(s)\left(g_0(s) - f_0(s)\right)^2}{(Qg_0(s)+(1-Q)f_0(s))^2} < 0,$$

as desired. ∎

A.3 Proof of Proposition 2

The proof of Lemma 1 implies that the direct effect satisfies

$$\frac{\partial \Delta(t|Q^*)}{\partial t}\Big|_{t=1-Q^*} > 0.$$

The first part of the result then follows immediately from Proposition 1.

Suppose that $\frac{d\Delta^*}{dQ}(Q) > 0$, and let $p = \Delta^*(Q)$. If $\Delta^*(Q') > p$ for all $Q' > Q$, then $Q \in \mathcal{Q}(p)$ and $1 \in \mathcal{Q}(p)$. Otherwise, $\Delta^*(\cdot)$ must equal p at some $Q' > Q$, in which case both Q and Q' are in $\mathcal{Q}(p)$. ∎

A.4   Proof of Proposition 3

If $\omega = 1$, then s has distribution

$$R_1(\cdot|Q) \equiv QG_1(\cdot) + (1 - Q)F_1(\cdot),$$

and if $\omega = 0$, then s has distribution

$$R_0(\cdot|Q) \equiv QG_0(\cdot) + (1 - Q)F_0(\cdot),$$

where $R_\omega(\cdot)$ has density $r_\omega(\cdot)$.

Suppose the service clarifies. Assumption 2 implies that

$$\frac{r_1(\cdot)}{r_0(\cdot)} = \frac{Qg_1(\cdot) + (1 - Q)f_1(\cdot)}{f_0(\cdot)}$$

is strictly increasing. Therefore, the receiver faces a monotone decision problem and so it suffices to show that if $\bar{Q} > Q$, then $R_\omega(\cdot|\bar{Q})$ is Lehmann more informative than $R_\omega(\cdot|Q)$. By the definition of Lehmann informativeness, it suffices to show that for any s,

$$R_1(s|Q) \geqslant R_1(s|\bar{Q})$$

with strict inequality for some s. But

$$R_1(s|Q) - R_1(s|\bar{Q}) = (Q - \bar{Q})\left(G_1(s) - F_1(s)\right) \geqslant 0,$$

with strict inequality for some s, because $Q < \bar{Q}$ and $G_1(\cdot)$ strictly dominates $F_1(\cdot)$ in the FOSD order by Assumption 2.

If the service embellishes, then the analogous argument requires that for any $\bar{Q} > Q$ and s,

$$R_0(s|Q) \geqslant R_0(s|\bar{Q}),$$

and strictly so for some s. But

$$R_0(s|Q) - R_0(s|\bar{Q}) = (Q - \bar{Q})(G_0(x) - F_0(x)) \geqslant 0,$$

and strictly so for some x, because $\bar{Q} > Q$ and $G_0(\cdot)$ strictly dominates $F_0(\cdot)$ in the FOSD order. ∎

A.5   Proof of Proposition 4

Given receiver beliefs Q^*, the agency's profit-maximizing price solves

$$\max_{p \geqslant 0}(p - c)(1 - t^*(p|Q^*)).$$

Since $c < \Delta(1|Q)$ and $\Delta(\cdot|Q)$ is continuous for any Q, the agency can earn strictly positive profits for any receiver beliefs Q. Since $c > \Delta(0|Q)$, whenever the agency earns strictly positive profits, $Q < 1$.

$t^*(p|Q)$ is differentiable in both of its arguments because $\frac{\partial \Delta}{\partial t}, \frac{\partial \Delta}{\partial Q} \neq 0$. The profit-maximizing price is therefore interior and satisfies the first-order condition

$$1 - t^*(p|Q^*) = (p - c)t_p^*(p|Q^*).$$

Beliefs are correct in equilibrium: $1 - t^*(p^*|Q^*) = Q^*$. So this expression can be rearranged to yield (4). ∎

A.6   Proof of Proposition 5

In the Bertrand game, the seller will either not buy the rhetorical service or buy from the agency with the lowest price, which we denote $\underline{p}$. Since $\Delta(1|Q) > c$, rhetorical services are purchased with positive probability in equilibrium. Because $\Delta(0|Q) \leqslant c$, a standard argument shows that $\underline{p} = c$ in equilibrium.

The maximum equilibrium purchase probability in the Bertrand game is $Q_C = \max\{Q|Q \in \mathcal{Q}(c)\}$, with corresponding marginal type t_C. Let Q_M be the purchase probability in an equilibrium of the monopolist game, with equilibrium price p_M and corresponding marginal type t_M. Since $\Delta(1|Q) > c \geqslant \Delta(0|Q)$ for all Q, $Q_C \in (0,1]$ and $Q_M \in (0,1)$, so (4) implies that $p_M > c$.

We argue that $Q_M < Q_C$. Suppose that $Q_M \geqslant Q_C$, so that $t_M \leqslant t_C$. Since $p_M > c$, it must be that $\Delta^*(Q_M) > \Delta^*(Q_C)$ in equilibrium. But $Q_C = \max\{Q|Q \in \mathcal{Q}(c)\}$, which means that for any $Q > Q_C$, $\Delta^*(Q) \neq c$. Now, $\Delta(t|Q)$ is continuous in its arguments and so $\Delta^*(\cdot)$ is continuous; since $\Delta^*(Q_M) > \Delta^*(Q_C)$, and $Q_M \geqslant Q_C$, the Intermediate Value Theorem requires that $\Delta^*(Q) > c$ for all $Q > Q_C$. But then $\Delta^*(1) > c$, which contradicts the assumption that $\Delta(0|Q) \leqslant c$ for any Q. We conclude that $Q_M < Q_C$, which by Proposition 3 implies that $u_R^*(Q_M) < u_R^*(Q_C)$ if the service clarifies and $u_R^*(Q_M) > u_R^*(Q_C)$ if the service embellishes.

To prove that competition can lead to lower utility for *all* players, we construct an example that satisfies Assumptions 1 and 2 except that $\frac{g_1(\cdot)}{g_0(\cdot)}$ is weakly rather than strictly increasing. We then argue that the result follows by a continuity argument. For this example, it is useful to describe the sender's utility as a function of the receiver's *posterior belief*, as defined by (5), rather than the likelihood ratio associated with that belief. To that end, define $\tilde{d}(\mu)$ as the receiver's optimal decision given belief μ and let $\tilde{u}_S(\mu, t) = u_S(\tilde{d}(\mu), t)$.

Suppose $c = 0$ and $\tilde{u}_S(\mu, t)$ is strictly convex in μ for all $t > 0$, with $\tilde{u}_S(\mu, 0) = 0$ for all μ. Let $f_1(\cdot) = g_1(\cdot)$ so that the service embellishes. Suppose that $\frac{f_1(\cdot)}{f_0(\cdot)}$ is strictly increasing but that $f_1(\cdot) = g_0(\cdot)$, so that s is uninformative about ω if (but only if) the sender purchases with probability 1.

For receiver beliefs $Q^* < 1$, $\mu^*(s|Q^*)$ is strictly increasing. Consequently, $\Delta(t|Q^*) > 0$ for all $t > 0$ and all types (except possibly $t = 0$) purchase the product in equilibrium. We conclude that $Q_C = 1$ in any equilibrium of the Bertrand market. As in the proof of Proposition 4, the monopolist earns strictly positive profit in any equilibrium, which implies

that $Q_M < 1$ because $\Delta^*(1) = 0$. The service embellishes, so $Q_M < Q_C$ immediately implies that the receiver is strictly better off in the monopolist game relative to the Bertrand game.

If $Q_C = 1$, then beliefs in the Bertrand game satisfy $\mu^*(s|1) = \gamma$ and so the sender earns $\tilde{u}_S(\gamma, t)$. In contrast, beliefs in the monopolist game are $\mu^*(s|Q_M)$, which is strictly increasing in s. Therefore, for any $t > 0$,

$$E_{s,\omega}\left[\tilde{u}_S(\mu^*(s|Q_M), t)\right] > \tilde{u}_S\left(E_{s,\omega}\left[\mu^*(s|Q_M)\right], t\right) = \tilde{u}_S(\gamma, t),$$

where the inequality follows because $\tilde{u}_S(\mu, t)$ is strictly convex in μ and the equality follows because beliefs are a martingale.

The agency earns strictly positive payoffs in the monopolist market and zero profit in the Bertrand market. A slight perturbation in $g_0(\cdot)$ does not change the fact that $Q_M < 1$ and $Q_C = 1$, so this argument continues to hold if $g_1(\cdot) \approx g_0(\cdot)$ but $\frac{g_1(\cdot)}{g_0(\cdot)}$ is strictly increasing. ∎

A.7 Proof of Proposition 6

Under the assumptions of the Proposition, $\mathcal{Q}(p)$ is a singleton on a neighborhood about p. Denote the unique element of $\mathcal{Q}(p)$ on this neighborhood by $Q^*(\cdot)$.

We first argue that $Q^*(\cdot)$ is decreasing, and strictly so if $Q^*(p) \in (0,1)$. Fix $p' < p$. If $Q^*(p) = 1$, then $\Delta^*(0) \geqslant p > p'$ and so $Q^*(p') = 1$, while $Q^*(p') \geqslant 0$ if $Q^*(p) = 0$. Otherwise, there exists t such that $\Delta^*(t) = p$. But $\Delta^*(\cdot)$ is continuous and $\Delta^*(t) = p > p'$, so either $\Delta^*(1) \geqslant p'$, in which case $Q^*(p') = 1 > Q^*(p)$, or there exists some $t' < t$ such that $\Delta^*(t') = p'$ by the Intermediate Value Theorem, in which case $Q^*(p') = 1 - t' > 1 - t = Q^*(p)$.

Consider a service that embellishes. For any $p < p_{PVT}$,

$$Q^*(p)(p - c) < \left(1 - t^*(p|Q^*(p_{PVT}))\right)(p - c) \leqslant Q^*(p_{PVT})(p_{PVT} - c),$$

where the first inequality holds because $t^*(\cdot|Q)$ is increasing in Q by Proposition 1 and the second inequality holds because p_{PVT} is a profit-maximizing price given beliefs $Q^*(p_{PVT})$.

Therefore, $p_{PUB} \geqslant p_{PVT}$. The agency earns strictly positive profit, so $Q^*(p_{PVT}) \in (0,1)$ because $\Delta(0|Q) \leqslant c$ for any Q. Consequently, $Q^*(p)$ is differentiable near p_{PVT} by the Implicit Function Theorem, since $\mathcal{Q}(\cdot)$ is always a singleton.

We argue that the agency would earn a strictly higher profit by increasing its price from p_{PVT}. To see this, note that $Q^*(p) = 1 - t^*(p|Q^*(p))$ and so

$$\frac{dQ^*}{dp} = -\frac{\partial t^*}{\partial p} - \frac{\partial t^*}{\partial Q}\frac{\partial Q^*}{\partial p} > -\frac{\partial t^*}{\partial p},$$

where the inequality follows because $\frac{\partial t^*}{\partial Q} > 0$ by Proposition 1 and $\frac{\partial Q^*}{\partial p} < 0$ by the argument above. Therefore,

$$\frac{dQ^*(p_{PVT})}{dp}(p_{PVT} - c) + Q^*(p_{PVT}) > -t_p^*(p_{PVT}|Q^*(p_{PVT}))(p_{PVT} - c) + Q^*(p_{PVT}) = 0,$$

while the equality follows because p_{PVT} solves (4). Combined with the result that $p_{PUB} \geqslant p_{PVT}$, we conclude that $p_{PUB} > p_{PVT}$.

Consider a service that clarifies. $\mathcal{Q}^*(p)$ is a singleton about p_{PVT}, which in particular means that $Q^*(p) < Q^*(p_{PVT})$ for any $p > p_{PVT}$. Therefore, $p \leqslant p_{PVT}$ by an argument similar to above. Since $\mathcal{Q}(p_{PVT}) \in (0,1)$ and $\frac{d\Delta^*(Q)}{dQ}|_{Q=Q^*(p_{PVT})} \neq 0$, $Q^*(\cdot)$ is differentiable on a neighborhood around p_{PVT} by the Implicit Function Theorem. So $p = p_{PVT}$ is not optimal for reasons similar to the above. We conclude that $p < p_{PVT}$ in this case. ∎

A.8   Proof of Proposition 7

Claim 1:   Given an equilibrium σ^* of the product design game, let $\mu_{\sigma^*}(s)$ be the receiver's posterior belief that the state is high conditional on signal s. Define

$$\mathcal{S}_M = \arg\max_s \mu_{\sigma^*}(s).$$

Then $\mathcal{S}_M$ is non-empty and $G_1(\mathcal{S}_M) = G_0(\mathcal{S}_M) = 1$.

Proof of Claim 1: Suppose that $\mathcal{S}_M$ is non-empty but that $G_\omega(\mathcal{S}_M) < 1$ for some $\omega \in \{0, 1\}$. Let $\bar{\mu} = \max_s \mu_{\sigma^*}(s)$, and consider the alternative product that sets $\tilde{G}_\omega(\mathcal{S}_M) = 1$ and is otherwise identical to G_ω. This perturbed product increases the sender's willingness-to-pay by

$$\int u_S^*(\mu_{\sigma^*}(s), t) \left[\gamma \left(d\tilde{G}_1 - dG_1 \right) + (1 - \gamma) \left(d\tilde{G}_0 - dG_0 \right) \right] =$$

$$(1 - \gamma G_1(\mathcal{K}_M) - (1 - \gamma)G_0(\mathcal{S}_M))u_S^*(\bar{\mu}) - \int_{s \notin \mathcal{S}_M} u_S^*(\mu_{\sigma^*}(s)) \left[\gamma dG_1 + (1 - \gamma)dG_0 \right] > 0$$

where the inequality follows because $\mu_{\sigma^*}(s) < \bar{\mu}$ for all $s \notin \mathcal{S}_M$, $\gamma \in (0, 1)$, and $G_\omega(\mathcal{S}_M) < 1$ for at least one ω. The agency can increase its price by this amount without affecting quantity sold, so such a deviation is profitable.

It remains to show that $\mathcal{S}_M$ is non-empty. If $\mathcal{S}_M$ is empty, then for any $G_\omega(\cdot)$, there exists an alternative $\tilde{G}_\omega(\cdot)$ that induces weakly higher posteriors everywhere and strictly higher posteriors with positive probability. Therefore, the agency would have a profitable deviation. ∎

Claim 2: An equilibrium of the product design game exists. In any equilibrium, $\mu_{\sigma^*}(s) = q$ on the support of the equilibrium signal distribution, and at least one of the following two conditions must hold: (i) $p = 0$, or (ii) the sender purchases the service with probability 1.

Proof of Claim 2: To prove existence, consider the following strategy profile: for $\omega \in \{0, 1\}$, $G_\omega(\cdot)$ is a uniform distribution over $[0, 1]$, $p = 0$, the sender purchases this service with probability 1, and $\mu_{\sigma^*}(s) = q$ for all $s \in [0, 1]$. Given $G_\omega(\cdot)$, $\mu_{\sigma^*}(\cdot)$ follows from Bayes' Rule. The sender is indifferent between purchasing or not and so is willing to purchase this service with probability 1. Any $G_\omega(\cdot)$ results in the same (degenerate) posterior distribution, so the sender is willing to pay 0 regardless of $G_\omega(\cdot)$ and the agency has no profitable deviation. Therefore, this strategy profile is an equilibrium of the product design game.

Consider any equilibrium σ^*. If $p > 0$ in this equilibrium, then the sender must purchase the service with probability 1; otherwise, the agency could deviate to $p - \epsilon$ with $\epsilon > 0$ and

generate a discrete increase in quantity sold, which would therefore be profitable for $\epsilon > 0$ sufficiently close to 0. But Claim 1 implies that $G_\omega(\cdot)$ induces the same posterior regardless of the true state in any equilibrium; since the sender purchases with probability 1, $\mu_{\sigma^*}(s) = q$ must hold on the support of $G_\omega(\cdot)$.

Suppose instead that $p = 0$ in this equilibrium. By claim 1, $G_\omega(\mathcal{S}_M) = 1$. Since $F_\omega(\cdot)$ has full support on $[0, 1]$, sender willingness-to-pay is strictly positive unless $\mu_{\sigma^*}(s) = q$ for all $s \in [0, 1]$, which proves the claim. ∎

A.9 Proof of Proposition 8

We can write

$$\Delta^r(t|Q) \equiv t \int_s l^r(s|Q) \left[\gamma(g_1(s) - f_1(s)) + (1 - \gamma)(g_0^r(s) - f_0^r(s)) \right] ds,$$

so

$$\frac{\partial \Delta^r(t|Q)}{\partial r} = t \int_s \left(\begin{array}{c} \frac{\partial l^r(s|Q)}{\partial r} \left[\gamma(g_1(s) - f_1(s)) + (1 - \gamma)(g_0^r(s) - f_0^r(s)) \right] + \\ (1 - \gamma)l^r(s|Q) \left[\frac{\partial g_0^r(s)}{\partial r} - \frac{\partial f_0^r(s)}{\partial r} \right] \end{array} \right) ds. \quad (6)$$

Now, $l^r(s|Q)$ satisfies strictly increasing differences in s and r, so $\frac{\partial l^r(s|Q)}{\partial r}$ is strictly increasing in s. Therefore,

$$\int_s \frac{\partial l^r(s|Q)}{\partial r} \left[\gamma(g_1(s) - f_1(s)) + (1 - \gamma)(g_0^r(s) - f_0^r(s)) \right] > 0 \quad (7)$$

by the strict version of Beesack's Inequality. If the service clarifies, then $g_0^r(\cdot) \equiv f_0^r(\cdot)$ and $\frac{\partial g_0^r}{\partial r} - \frac{\partial f_0^r}{\partial r} = 0$, in which case (6) is strictly positive, proving the claim.

Suppose the service does not clarify. Since $\int_s (g_0^r(s) - f_0^r(s))ds \equiv 0$ for all r, $\int_s \left(\frac{\partial g_0^r(s)}{\partial r} - \frac{\partial f_0^r(s)}{\partial r} \right) = 0$. Moreover, $l^r(s|Q)$ is strictly increasing in s by Assumption 2. Therefore, if $\frac{\partial g_0^r(s)}{\partial r}$ single-

crosses $\frac{\partial f_0^r(s)}{\partial r}$ from below, then the strict version of Beesack's Inequality implies that

$$\int_s l^r(s|Q) \left[\frac{\partial g_0^r(s)}{\partial r} - \frac{\partial f_0^r(s)}{\partial r} \right] ds > 0.$$

Together with (7), we conclude that (6) is again strictly positive. ∎

A.10   Proof of Proposition 9

The posterior belief $\mu^*(\cdot|0)$ is strictly increasing by Assumption 2. Consider the following strategy profile: for every $\theta \in \Theta$, a sender with ability θ makes argument $(1, \theta_T)$ if $\omega = 1$ and $(1, \theta_L)$ if $\omega = 0$. A belief system consistent with this strategy profile is: the receiver's posterior equals $\mu^*(s|0)$ if he observes $(1, s)$ and equals 0 if he observes $(0, s)$ for any $s \in \mathbb{R}$. Any deviation from this strategy profile to $a = (0, s)$ induces posterior belief 0 and so cannot be profitable. To be feasible, a deviation to $(1, s)$ must satisfy $s < \theta_T$ if $\omega = 1$ or $s < \theta_L$ if $\omega = 0$. But these deviations are not profitable because $\mu^*(\cdot|0)$ is increasing. So the specified strategy profile is an equilibrium, which proves that an equilibrium with the desired properties exists.

Next, we show that this equilibrium induces the most informative equilibrium mapping from ω to a in the Lehmann sense. Fix a PBE, and let $\mu(a)$ be the receiver's posterior belief that the state is high after observing a. Without loss, assume that $\mu(a) = 0$ whenever a is not sent on the equilibrium path. Define

$$\mu_0(s) = \sup_{k' \leqslant k} \mu(0, s)$$

and

$$\mu_1(s) = \sup_{k' \leqslant k} \mu(1, s).$$

Note that $\mu_0(\cdot)$ and $\mu_1(\cdot)$ are both increasing. In equilibrium, the sender induces posterior $\mu_1(\theta_T)$ if $m = 1$ and $\mu_0(\theta_L)$ if $m = 0$.

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We first argue that if $a = (1, s)$ is sent on-path, then $\mu_1(s) \geqslant \mu_0(s)$. Suppose $(1, s)$ is on-path and $\mu_1(s) < \mu_0(s)$. If $\omega = 0$, then any sender who can send $(1, s')$ with $s' \leqslant s$ can also send $(0, s')$ with $s' \leqslant s$. Therefore, no sender chooses $a = (1, s)$ if $\omega = 0$. But $(1, s)$ is sent on the equilibrium path, so it must be sent only if $\omega = 1$ and so $\mu(1, s) = 1 \geqslant \mu_0(s)$; contradiction.

Next, we claim that if there exists $s' \leqslant s$ such that both $(1, s)$ and $(0, s')$ are on-path, then $\mu_1(s) = \mu_0(s)$ without loss. Suppose there exists $s' \leqslant s$ such that $(0, s')$ is on-path as well. If $\mu_1(s) > \mu_0(s)$, then no sender chooses $(0, s')$ with $s' \leqslant s$ if $\omega = 1$. So any on-path $(0, s')$ satisfies $\mu(0, s') = 0$, and moreover, it must be the case that $\mu_1(0) = 0$ in order for $(0, s)$ to be on-path. But then we can perturb the equilibrium so that all senders who make argument $(0, s')$ with $s' \leqslant s$ instead choose $(1, 0)$ without affecting equilibrium persuasion. In this perturbed equilibrium, $\mu_1(s) = \mu_0(s)$ whenever $(1, s)$ is on-path and there exists a $s' \leqslant s$ such that $(0, s')$ is on-path.

Now, define

$$\mathcal{S}_1(s) \equiv \{s' | \mu_1(s') = \mu_1(s)\}$$

and

$$\mathcal{S}_0(s) \equiv \{s' | \mu_0(s') = \mu_0(s)\}.$$

Then let

$$\mathcal{S}(s) \equiv \mathcal{S}_1(s) \cap \mathcal{S}_0(s),$$

and note that $s \in \mathcal{S}(s)$ and so $\mathcal{S}(\cdot)$ is nonempty on its domain. Moreover, for each $\mathcal{S}(s)$, there exists some $s' \in \mathcal{S}(s)$ such that either $(1, s')$ or $(0, s')$ is on the equilibrium path.

Fixing the sender's strategy, suppose that rather than observing $a = (m, s)$, the receiver instead observes $\mathcal{S}(s)$. Clearly, this alternative information structure results in the receiver having weakly less information. We claim that it results in the receiver having *exactly the same* amount of information. Indeed, for each $\mathcal{S}(s)$, one of three possibilities obtains.

First, it might be that for all $s' \in \mathcal{S}(s)$, $a = (0, s')$ is off-path and so the receiver infers

posterior $\mu_1(s')$ when observing $\mathcal{S}(s)$. Second, it might be that for all $s' \in \mathcal{S}(s)$, $a = (1, s')$ is off-path and so the receiver infers posterior $\mu_0(s')$ when observing $\mathcal{S}(s)$. Finally, there might exist $s', s'' \in \mathcal{S}(s)$ such that $(1, s')$ and $(0, s'')$ are both on-path. If $s'' \leqslant s'$, then $\mu_1(s) = \mu_0(s)$ by the argument above and so the receiver infers posterior $\mu(1, s)$ when observing $\mathcal{S}(s)$. If $s'' > s'$, then $\mathcal{S}(s) \subsetneq \mathcal{S}_0(s)$ because $\mu_0^*(s)$ can change only if $(0, s)$ is sent on-path. But then either $\mu_0^*(s) = 0$ for all $s \in \mathcal{S}(s)$, in which case the receiver can infer $\mu(1, s)$ when observing $\mathcal{S}(s)$, or $\mu_0(s) > 0$ for all $s \in \mathcal{S}(s)$, in which case there exists some $\hat{s} < s'$ with $\hat{s} \in \mathcal{S}_0(s) \backslash \mathcal{S}(s)$ such that $(0, \hat{s})$ is sent on-path. But then $\mu_0(\hat{s}) = \mu_1(s)$, implying $\mu_1(s) = \mu_0(s)$ because $\mu_0(\cdot)$ is constant on $\mathcal{S}_0(s)$ and $\hat{s} \in \mathcal{S}_0(s)$.

We have established that we can treat the sender as communicating $\mathcal{S}(s)$ rather than the argument (m, s) in equilibrium. By the argument above, the posterior induced by $\mathcal{S}(s)$ equals $\mu_1(s)$ if there exists a $k' \in \mathcal{S}(s)$ such that $(1, s')$ is on-path and equals $\mu_0(s)$ otherwise. Next, we claim that the posteriors induced by $\mathcal{S}(s)$ are increasing in s. Suppose not. Note that $\mathcal{S}(\cdot)$ partitions $\mathbb{R}_+$, and suppose $s < s'$ are such that $\mathcal{S}(s')$ induces a strictly lower posterior than $\mathcal{S}(s)$. This cannot be the case if $m = 1$ is on-path in both $\mathcal{S}(s)$ and $\mathcal{S}(s')$, since $\mu_1(s)$ is increasing in s. Similarly if $m = 1$ is *not* on-path in both $\mathcal{S}(s)$ and $\mathcal{S}(s')$.

Suppose $m = 1$ is on-path in $\mathcal{S}(s')$ but not in $\mathcal{S}(s)$, and $\mu_0(s) > \mu_1(s')$. Then $\mathcal{S}(s')$ is sent only if $\omega = 1$, and so $\mu_1(s') = 1$; contradiction. If instead $m = 1$ in on-path in $\mathcal{S}(s)$ but not in $\mathcal{S}(s')$, and $\mu_1(s) > \mu_0(s')$. Then $\mathcal{S}(s')$ is sent only if $\omega = 0$ and so $\mu_1(s') = 0$. But then without loss, $\mathcal{S}(s')$ is never sent when $\omega = 0$ either; contradiction. So beliefs are increasing in $\mathcal{S}(s)$. One implication of this result is that we can construct an information structure that is equivalent to $\mathcal{S}(\cdot)$ with a density that satisfies MLRP.[17]

If $\omega = 1$, then $\sup \mathcal{S}(s) \leqslant \theta_T$ because only $s \leqslant \theta_T$ are feasible. If $\omega = 0$, then $\sup \mathcal{S}(s) \geqslant \theta_L$; otherwise, the sender could induce a strictly higher posterior by making some argument with $s \in \mathcal{S}(\theta_L)$. Now, consider the alternative information structure in which the sender

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[17]For example, consider the signal structure in which, whenever the receiver would observe $\mathcal{K}(k)$, they instead observe some $k' \in \mathcal{K}(k)$ drawn uniformly at random from $\mathcal{K}(k)$. This equivalent signal structure has a distribution that is differentiable almost everywhere and satisfies MLRP.

chooses $\mathcal{S}(\theta_T)$ if $\omega = 1$ and $\mathcal{S}(\theta_L)$ if $\omega = 0$. This alternative information structure has the property that posteriors are increasing in $\mathcal{S}(s)$ because $\frac{f_1(\cdot)}{f_0(\cdot)}$ is increasing. Moreover, if $\omega = 1$, then the sender chooses a weakly higher $\mathcal{S}(\cdot)$; if $\omega = 0$, then the sender chooses a weakly lower $\mathcal{S}(\cdot)$. Hence, this alternative information structure entails in a FOSD shift upwards of the marginal distribution over $\mathcal{S}(\cdot)$ if $\omega = 1$, and a FOSD shift downwards of the marginal distribution over $\mathcal{S}(\cdot)$ if $\omega = 0$. Consequently, it is Lehmann more informative.

The garbling $s \to \mathcal{S}(s)$ transforms the information conveyed in the earnest equilibrium to that of this alternative information structure. Therefore, the equilibrium in the statement of the result leads to a Blackwell more informative signal distribution than this alternative. Hence, the desired equilibrium is Lehmann more informative than any other PBE. ∎

B   Exploring the Communication Game

B.1   What if $F_\omega(\cdot)$ Doesn't Satisfy MLRP?

The paper assumes that $F_\omega(\cdot)$ satisfies strict MLRP, which implies that $\mu^*(\cdot|0)$ is strictly increasing. The communication game in Section 6 does not rely on this assumption, and indeed, we can partially characterize the patterns of communication that emerge if $F_\omega(\cdot)$ does not satisfy it.

Unlike Proposition 9, the equilibria of the communication game are not clearly welfare-ranked if $\mu^*(\cdot|0)$ is non-increasing. To discipline our analysis, we propose an equilibrium refinement that generates a natural interpretation of the strength of an argument. We say a PBE is an **earnest equilibrium** if the sender chooses $a = (1, \theta_T)$ with probability 1 whenever $\omega = 1$. This refinement rules out two phenomena in equilibrium: a strong argument might never be made if it is assigned a low (off-path) posterior, or the sender might reverse the meaning of the message by choose $m = 0$ when $\omega = 1$ and $m = 1$ when $\omega = 0$. Note that the outcome characterized in Proposition 9 can be attained in an earnest equilibrium.

The first step in this analysis is to state a definition. Let

$$z(\cdot|[\underline{s}, \bar{s}]) = \frac{\gamma f_1(\cdot) + (1-\gamma) f_0(\cdot)}{\gamma \left(F_1(\bar{s}) - F_1(\underline{s}) \right) + (1-\gamma) \left(F_0(\bar{s}) - F_0(\underline{s}) \right)}$$

be the conditional density function for arguments on $[\underline{s}, \bar{s}]$ if the sender chooses $a = (1, \theta_T)$ if $\omega = 1$ and $a = (1, \theta_L)$ if $\omega = 0$.

Definition 1 *Let Assumption 1 hold. An increasing function $\hat{\mu} : \mathbb{R}_+ \to [0, 1]$ is a* **candidate posterior** *if for any $s \in \mathbb{R}_+$, either $\hat{\mu}(s) = \mu^*(s|0)$ or $\hat{\mu}(\cdot)$ is constant on a closed interval that contains s. Let $I \subseteq [0, 1]$ be an interval such that $\hat{\mu}(\cdot)$ is constant on I but not on any other interval in $\mathbb{R}_+$ that contains I. Letting $\underline{s} = \inf I$, for any $s \in I$,*

$$\hat{\mu}(s) \equiv \mu^* \leqslant \int_{\underline{s}}^{s} \mu^*(x|0) z(x|[\underline{s}, s]) dx, \tag{8}$$

where (8) holds with equality as $s \uparrow \sup I$.

The next lemma demonstrates a tight connection between equilibrium persuasion in an earnest equilibrium and a candidate posterior. Since candidate posteriors must satisfy the restrictions listed in Definition 1, this result puts considerable structure on earnest equilibrium play.

Lemma 2 *Let Assumption 1 hold. In any earnest equilibrium, there exists a candidate posterior $\hat{\mu}(\cdot)$ such that for any realization of $\theta \in \Theta$, the receiver's equilibrium posterior belief that $\omega = 1$ equals $\hat{\mu}(\theta_T)$ if $\omega = 1$ and $\hat{\mu}(\theta_L)$ if $\omega = 0$.*

Proof of Lemma 2

Let σ^* be an earnest equilibrium of a smooth game. Define $\tilde{\mu}(a)$ as the receiver's posterior belief that $\omega = 1$ following argument a. Because F_1 has full support on $\mathbb{R}_+$, for any $s \in \mathbb{R}_+$, $(1, s)$ is sent with positive probability. But then $\tilde{\mu}(1, s)$ must be weakly increasing in s: if $\tilde{\mu}(1, s') < \tilde{\mu}(1, s)$ for $s' > s$, then a sender with $\theta_T = s'$ and $\omega = 1$ would have a feasible and profitable deviation to $a = (1, s)$.

By definition, if $\omega = 1$, then $m = 1$. Therefore, any $a = (0, s)$ that is sent on the equilibrium path satisfies $\tilde{\mu}(a) = 0$ and so either $m = 0$ is never sent on the equilibrium path, or there exists some s such that $\tilde{\mu}(1, s) = 0$. In the former case, we can equivalently define the receiver's posterior $\hat{\mu}(s) \equiv \tilde{\mu}(1, s)$. In the latter case, $\tilde{\mu}(1, 0) = 0$ because $\tilde{\mu}((1, \cdot))$ is weakly increasing, so we can interpret any argument $a = (0, s)$ as the argument $a = (1, 0)$, and thereby similarly define the receiver's posterior as a function of the argument's strength alone.

Now, $\hat{\mu}(s)$ must be increasing because $\tilde{\mu}(1, \cdot)$ is increasing. So in equilibrium, a sender with type θ must induce posterior $\hat{\mu}(\theta_T)$ if $\omega = 1$ and $\hat{\mu}(\theta_L)$ if $\omega = 0$, since otherwise she would have a profitable deviation to a stronger feasible argument. Hence, it remains to show that $\hat{\mu}(\cdot)$ is a candidate posterior.

We have already shown that $\hat{\mu}(\cdot)$ is increasing. Suppose there exists some $s \in \mathbb{R}_{++}$ such that $\hat{\mu}(s) \neq \mu^*(s|0)$. If $\omega = 1$, then $a = (1, s)$ if and only if $\theta_T = s$. If $\omega = 0$ leads to $a = (1, s)$ if and only if $\theta_L = s$, then $\hat{\mu}(s) = \mu^*(s|0)$ by definition of $\mu^*(\cdot|0)$. Therefore, either (i) the sender does not choose $a = (1, s)$ with probability 1 if $\omega = 0$ and $\theta_L = s$, or (ii) a sender with $\theta_L \neq s$ makes argument $a = (1, s)$ with positive probability if $\omega = 0$ (or both).

In case (i), the sender must instead make another feasible argument: either $(1, s')$ for some $s' < s = \theta_L$, or $(0, s')$ for some $s' \leqslant \theta_T$. If the former, then the sender has the incentive to do so only if $\hat{\mu}(s') = \hat{\mu}(s)$ because $\hat{\mu}(\cdot)$ is increasing. If the latter, then $\hat{\mu}(s) = 0$ by the argument above, which implies that $\hat{\mu}(s') = 0$ for all $s' \in [0, s]$. So $\hat{\mu}(\cdot)$ is constant on some non-empty interval $[s', s]$.

In case (ii), the sender could have instead made argument $(1, \theta_L)$. For $(1, s)$ to be a feasible argument for the sender, $\theta_L > s$, and so it must be that $\hat{\mu}(\cdot)$ is constant on the non-empty interval $[s, \theta_L]$. So $\hat{\mu}(\cdot)$ is flat on a non-empty interval interval about s whenever it does not coincide with $\mu(\cdot)$.

Finally, let $(\underline{s}, \bar{s})$ be such that $\hat{\mu}(\cdot)$ is constant on this interval but not on any open interval that contains $(\underline{s}, \bar{s})$. Every argument $(1, \tilde{s})$ with $\tilde{s} \in (\underline{s}, \bar{s})$ induces the same posterior, which we denote by μ^*. Therefore, for any $s \in (\underline{s}, \bar{s})$,

$$\mu^* = \frac{q\mathrm{Pr}_{\sigma^*}\{a = (1, x) \text{ for } x \in (\underline{s}, s)|\omega = 1\}}{q\mathrm{Pr}_{\sigma^*}\{a = (1, x) \text{ for } x \in (\underline{s}, s)|\omega = 1\} + (1-q)\mathrm{Pr}_{\sigma^*}\{a = (1, x) \text{ for } x \in (\underline{s}, s)|\omega = 0\}}.$$

In any earnest equilibrium,

$$\mathrm{Pr}_{\sigma^*}\{a = (1, x) \text{ for } x \in (\underline{s}, s)|\omega = 1\} = F_1(s) - F_1(\underline{s}).$$

Furthermore, any sender with $\theta_L \in (\underline{s}, s)$ cannot make an argument stronger than s and is unwilling to make an argument that is weaker than $\underline{s}$, since such an argument would induce

a strictly lower posterior by definition of $\underline{s}$. Therefore,

$$\Pr_{\sigma^*}\{a = (1, x) \text{ for } x \in (\underline{s}, s)|\omega = 0\} \geqslant F_0(s) - F_0(\underline{s}). \tag{9}$$

Consequently,

$$\begin{aligned}
\mu^* &\leqslant \frac{q(F_1(s)-F_1(\underline{s}))}{q(F_1(s)-F_1(\underline{s}))+(1-q)(F_0(s)-F_0(\underline{s}))} \\
&= \int_{\underline{s}}^{s} \mu^*(x|0)z(x|[\underline{s}, s])dx,
\end{aligned}$$

where the equality follows from the definitions of $\mu^*(x|0)$ and $z(x|[\underline{s}, s])$. So (8) must hold for every $s \in (\underline{s}, \bar{s})$. For $s = \bar{s}$, (9) must hold with equality because no sender with $\theta_L > \bar{s}$ is willing to make an argument with strength in $[\underline{s}, \bar{s}]$. Therefore, the steps given above imply that (8) holds with equality for $s = \bar{s}$. ∎

Lemma 2 shows that every earnest equilibrium corresponds to a candidate posterior. The equilibrium mapping from signals to posterior beliefs takes a relatively simple form: for any type θ, the receiver's posterior equals $\hat{\mu}(\theta_T)$ if $\omega = 1$ and $\hat{\mu}(\theta_L)$ if $\omega = 0$, where $\hat{\mu}(\cdot)$ either coincides with $\mu^*(\cdot|0)$ or is flat on an interval and coincides with the *expectation* of $\mu^*(\cdot|0)$ on that interval.

Our next result uses Lemma 2 to characterize earnest equilibrium persuasion for the case where $\mu^*(\cdot|0)$ is decreasing over at most one interval. We show that the resulting candidate posterior is an "ironed" version of $\mu^*(\cdot|0)$ that flattens out the decreasing region. In principle, it should be relatively straightforward to extend this ironing argument to more complicated $\mu^*(\cdot|0)$, but the proof would be complicated and quite tedious and so we do not attempt it here.

Proposition 10 *Let Assumption 1 hold, and suppose there exist $s_L, s_H \in \mathbb{R}_+$ with $s_L < s_H$ such that $\mu^*(s|0)$ is strictly increasing for $s < s_L$ and $s > s_H$ and strictly decreasing on $s \in (s_L, s_H)$. Then a unique candidate posterior and a corresponding earnest equilibrium both exist. The candidate posterior, $\hat{\mu}(\cdot)$, is constant on an interval $\mathcal{I}$ with $[s_L, s_H] \subseteq \mathcal{I}$ and otherwise satisfies $\hat{\mu}(s) = \mu^*(s|0)$.*

Proof of Proposition 10

By Lemma 2, it suffices to show that there exists a unique candidate posterior. Let $\hat{\mu}(\cdot)$ be a candidate posterior. We claim that there exists an interval $\mathcal{I} \subseteq [0,1]$ such that $[s_L, s_H] \subseteq \mathcal{I}$ and $\hat{\mu}(\cdot)$ is constant on $\mathcal{I}$.

Suppose not. Then there exist $\underline{s} < \bar{s}$ such that $\underline{s}, \bar{s} \in [s_L, s_H]$ and $\hat{\mu}(\underline{s}) < \hat{\mu}(\bar{s})$. Because $\mu^*(\cdot|0)$ is decreasing on $[s_L, s_H]$, we can choose $\underline{s}$ and $\bar{s}$ to be part of intervals $\underline{I}$ and $\bar{I}$, respectively, where $\hat{\mu}(\cdot)$ is constant on both $\underline{I}$ and $\bar{I}$ and $\sup \underline{I} \leqslant \inf \bar{I}$. Now, $\hat{\mu}(\bar{s}) \leqslant \mu^*(\inf \bar{I}|0)$, since otherwise (8) would be violated on a small enough sub-interval of $\bar{I}$ because $\mu^*(\cdot|0)$ is continuous. But $\mu^*(\inf \bar{I}|0) \leqslant \mu^*(\sup \underline{I}|0)$ because $\mu^*(\cdot|0)$ is decreasing on $[s_L, s_H]$, so $\hat{\mu}(\underline{s}) < \mu^*(\sup \underline{I}|0)$. If $\hat{\mu}(\underline{k}) \leqslant \mu^*(\inf \underline{I}|0)$, then $\hat{\mu}(\cdot)$ lies everywhere below $\mu^*(\cdot|0)$ on $\underline{I}$ because $\mu^*(\cdot|0)$ is increasing and then decreasing on $\underline{I}$, and so (8) cannot hold with equality on $\underline{I}$. If $\hat{\mu}(\underline{s}) > \mu^*(\inf \underline{I}|0)$, then (8) is violated for a sufficiently small sub-interval of $\underline{I}$. So $\hat{\mu}(\cdot)$ must be constant on some interval $\mathcal{I} \supseteq [k_L, k_H]$.

Define $\bar{s} = \sup \mathcal{I}$ and $\underline{s} = \inf \mathcal{I}$, so that the candidate posterior equals some constant $\hat{\mu}$ on $\mathcal{I}$. We claim that $\hat{\mu}(\bar{s}) = \mu^*(\bar{s}|0)$. Suppose $\hat{\mu}(\bar{s}) < \mu^*(\bar{s}|0)$. The candidate posterior is increasing, so $\hat{\mu} < \mu^*(\bar{s}|0)$. But (8) holds with equality on $[\underline{s}, \bar{s}]$ and $\mu^*(\cdot|0)$ is continuous, so (8) must be violated on $[\underline{s}, \bar{s} - \epsilon]$ for $\epsilon > 0$ sufficiently small. Hence, $\hat{\mu} \geqslant \mu^*(\bar{s}|0)$. Moreover, $\hat{\mu}(\bar{s} + \epsilon) > \mu^*(\bar{s} + \epsilon|0)$ for any sufficiently small $\epsilon > 0$ because $\hat{\mu}(\cdot)$ is increasing and $\mu^*(\cdot|0)$ is continuous. But then $\hat{\mu}(\cdot)$ must be constant on some interval near $\bar{s} + \epsilon$, I_ϵ, where $\inf I_\epsilon \geqslant \bar{s}$ and so $\hat{\mu}(\inf I_\epsilon) > \mu^*(\inf I_\epsilon|0)$. Hence, $\hat{\mu}(\cdot)$ violates (8) on a small enough sub-interval of I_ϵ. So $\hat{\mu} = \mu^*(\bar{s}|0)$.

Next, we claim that $\hat{\mu} \leqslant \mu^*(\underline{s}|0)$, with equality unless $\underline{s} = 0$. If $\hat{\mu} > \mu^*(\underline{s}|0)$, then (8) is violated on $[\underline{s}, \underline{s} + \epsilon]$ for sufficiently small $\epsilon > 0$. If $\hat{\mu} < \mu^*(\underline{s}|0)$ and $\underline{s} > 0$, then $\hat{\mu}(\underline{s} - \epsilon) < \mu^*(\underline{s} - \epsilon|0)$ for any $\epsilon > 0$ sufficiently small. Therefore, there exists an interval I_ϵ such that $\underline{s} - \epsilon \in I_\epsilon$ and $\hat{\mu}(\cdot)$ is constant on I_ϵ. But $\sup I_\epsilon \leqslant \underline{s}$ and so $\mu^*(\cdot|0)$ is strictly increasing on I_ϵ. Therefore, either $\hat{\mu}(s) < \mu^*(s|0)$ for almost all $s \in I_\epsilon$, in which case (8) cannot hold with equality on I_ϵ, or $\hat{\mu}(\underline{k} - \epsilon) > \mu^*(\inf I_\epsilon|0)$, in which case (8) is violated on a

sufficiently small sub-segment of I_ϵ. So $\hat{\mu} \leqslant \mu^*(\underline{s}|0)$, with equality unless $\underline{s} = 0$.

$\mu^*(\cdot|0)$ is strictly increasing on $[0,1]\backslash\mathcal{I}$. Therefore, $\hat{\mu}(\cdot)$ cannot be constant on any interval $\tilde{I} \in [0,1]\backslash I$ without violating (9). Consequently, $\hat{\mu}(\cdot)$ must be continuous and coincide with $\mu^*(\cdot|0)$ except on a single interval at which it is constant. To satisfy (8), this constant region must cross $\mu^*(\cdot|0)$. Therefore, for any candidate posterior, there exists some $\tilde{s} \in [s_L, s_H]$ such that the candidate posterior equals

$$
\hat{\mu}(s) \equiv \begin{cases} \min\left\{\mu^*(s|0), \mu^*(\tilde{s}|0)\right\} & s < s_L \\[2mm] \mu^*(\tilde{s}|0) & s \in [s_L, s_H] \\[2mm] \max\left\{\mu^*(s|0), \mu^*(\tilde{s}|0)\right\} & s > s_H \end{cases} \;.
$$

Finally, we argue that there exists exactly one $\tilde{s} \in [s_L, s_H]$ such that $\hat{\mu}(\cdot)$ is a candidate posterior. Letting $[\underline{s}(\tilde{s}), \bar{s}(\tilde{s})]$ be the interval on which $\hat{\mu}(\cdot)$ is constant, consider

$$
\mu^*(\tilde{s}|0) - \int_{\underline{s}(\tilde{s})}^{\bar{s}(\tilde{s})} \mu^*(x|0) z(x|[\underline{s}(\tilde{s}), \bar{s}(\tilde{s})]) dx.
$$

Since $\hat{\mu}(\cdot)$ is a candidate posterior, this expression equals 0 by (8). Since $\hat{\mu}(\cdot) = \mu^*(\cdot|0)$ everywhere outside the interval $[\underline{s}(\tilde{s}), \bar{s}(\tilde{s})]$, this expression has the same sign as

$$
\int_{\mathbb{R}_+} \left(\hat{\mu}(x) - \mu^*(x|0)\right) z(x|\mathbb{R}_+) dx. \tag{10}
$$

Now, $\mu^*(\cdot|0)$ is decreasing and continuous on $[s_L, s_H]$. Further, $\hat{\mu}(\cdot) \geqslant \mu^*(\cdot|0)$ if $\tilde{s} = s_L$ and $\hat{\mu}(\cdot) \leqslant \mu^*(\cdot|0)$ if $\tilde{s} = s_H$. Therefore, (10) is strictly positive at $\tilde{s} = s_L$, strictly negative at $\tilde{s} = s_H$, and strictly decreasing in $\tilde{s}$. So (10) equals 0 for exactly one $\tilde{s}$. But then the unique candidate posterior in this game equals $\hat{\mu}(\cdot)$ for this $\tilde{s}$.

It remains to argue that there exists an earnest equilibrium of the game. The equilibrium posterior must be given by the unique candidate posterior $\hat{\mu}(\cdot)$. The definition of an earnest equilibrium pins down the sender's strategy if $\omega = 1$. Consider the following strategy if $\omega = 0$.

If θ_L is such that $\hat{\mu}(\theta_L) \leqslant \mu^*(\theta_L|0)$, then $a = (1, \theta_L)$. If θ_L is such that $\hat{\mu}(\theta_L) > \mu^*(\theta_L|0)$, then $a = (1, \theta_L)$ with probability $\alpha(\theta_L) < 1$ defined by

$$\frac{\gamma f_1(\theta_L)}{\gamma f_1(\theta_L) + (1-\gamma)\alpha(\theta_L)f_0(\theta_L)} = \hat{\mu}(\theta_L)$$

With the complementary probability, the sender chooses an argument with a strength in $\mathcal{S} = \{s \in [\underline{s}(\tilde{s}), \bar{s}(\tilde{s})] | \hat{\mu}(s) < \mu^*(s|0)\}$. Every argument in this set is feasible because $\mu^*(\cdot|0)$ is decreasing over exactly one interval, so any s with $\hat{\mu}(s) > \mu^*(s|0)$ is strictly larger than any s with $\hat{\mu}(s) < \mu^*(s|0)$. The distribution of arguments over $\mathcal{S}$ is chosen (independent of the sender's type) so that the posterior equals $\hat{\mu}(s)$ at every $s \in \mathcal{S}$. Since (10) holds with equality on $[\underline{s}(\tilde{s}), \bar{s}(\tilde{s})]$, such a distribution exists. No sender has a profitable deviation from this strategy by construction, so it is an earnest equilibrium. ∎

B.2 The Set of PBE in a Simple Example

This appendix characterizes the full set of Perfect Bayesian Equilibrium payoffs in a simple example of the communication model from Section 6. This analysis shows how the refinement to earnest equilibrium constrains the set of equilibrium payoffs in an example. Note that this example does not satisfy Assumption 1.

Suppose that sender's rhetorical ability is one of three types, $\Theta = \{(1, 0), (1, 1), (2, 1)\}$, where $F(\cdot)$ has full support on Θ. We refer to a sender with ability $(1, 0)$, $(1, 1)$, and $(2, 1)$ as a **normal**, a **BSer**, and an **orator**, respectively. Let $\rho(\theta)$ be the probability that $\theta \in \Theta$ is realized, with marginals $\rho^T(\cdot)$ over θ_T and $\rho^L(\cdot)$ over θ_L. In this example, $2 > \max_{\theta \in \Theta} \theta_L$, so an argument with $s = 2$ can be made only if $\omega = 1$ and so reveals that the state is high. More generally, the stronger an argument, the more likely it is to be the maximum feasible argument under truth-telling in this example. Therefore, by an argument analogous to that of Proposition 9, there exists an essentially unique earnest equilibrium in this setting.

Proposition 11 *In this example, define*

$$\mu^* = \frac{q\rho^T(1)}{q\rho^T(1) + (1-q)\rho^L(1)} \in (0,1).$$

In any earnest equilibrium, high-state orators choose $a = (1,2)$ and induce posterior $\mu = 1$, low-state orators, low-state BSers, and high-state normals choose $a = (1,1)$ and induce posterior $\mu = \mu^*$, and low-state normals induce posterior $\mu = 0$.

Proof: Suppose $\omega = 0$. If $\theta = (1,0)$, then the sender's feasible arguments are $\{(1,0)\} \cup \{(0,k)|k \leqslant 1\}$, none of which are sent if $\omega = 1$. So a normal must induce posterior $\mu = 0$ in any earnest equilibrium. If $\rho^T(1) > 0$, then $a = (1,1)$ induces a strictly positive posterior in equilibrium, while any $a \in \{(1,s)|s < 1\} \cup \{(0,s)|s \leqslant 2\}$ induces posterior 0. Therefore, the orator and BSer must both make argument $a = (1,1)$ when $\omega = 0$, which induces posterior μ^* in equilibrium.

It is straightforward to show that no sender has a profitable deviation from this strategy. So every earnest equilibrium must satisfy the desired properties. ∎

The essentially unique earnest equilibrium in this setting conforms closely to the earnest equilibria in smooth games with increasing $\mu(\cdot)$. Next, we show that there exists an essentially unique PBE that is **not** earnest in this setting. To do so, we impose the (mild) condition that if $\theta_L < s$ for all θ in the support of $F(\cdot)$, then the receiver's posterior if he observes $a = (m,s)$ equals $\Pr\{\omega = m|a\} = 1$.

Proposition 12 *In this example, define*

$$\bar{\mu} = \frac{(1 - \rho(2,1))q}{1 - \rho(2,1)q} \in (0,q).$$

In any PBE satisfying $\Pr\{\omega = 1|(1,2)\} = 1$, either (i) the equilibrium mapping from (θ,ω) to the receiver's posterior belief is identical to an earnest equilibrium, or (ii) the receiver's posterior belief equals $\bar{\mu}$ unless $\theta = (2,1)$ and $\omega = 1$, in which case $a = (1,2)$ and $\mu = 1$.

Proof: Any $a = (m, s)$ with $s \in (1, 2]$ can be sent only if $\theta = (2, 1)$ and $\omega = m$, and similarly any $a = (m, s)$ with $s \in (0, 1]$ can be sent only if $\theta \in \{(1, 1), (2, 1)\}$ or $\theta = (1, 0)$ and $\omega = m$. Therefore, for the purposes of identifying equilibrium beliefs, we can restrict attention to equilibria in which any on-path arguments satisfy $s \in \{0, 1, 2\}$.

By our definition of PBE, $a = (1, 2)$ must induce posterior $\mu = 1$ in any equilibrium. Let $\{\mu^1, ..., \mu^L\}$ be the set of posteriors induced on the equilibrium path, where $\mu^1 < ... < \mu^L$. If $\omega = 1$ and $\theta = (2, 1)$, then $a = (1, 2)$ is feasible and induces posterior $\mu = 1$. So $\mu^L = 1$ and hence $L > 1$. Any other argument is feasible when $\omega = 0$ for at least one ability type, so no other argument can induce posterior $\mu = 1$.

If $\omega = 1$ but $\theta \neq (2, 1)$, then $a = (1, 2)$ and hence $\mu^{L-1} > 0$. If $\omega = 0$ and $\theta = (2, 1)$, then the sender can make the argument that induces μ^{L-1}, so she never chooses $a = (0, 2)$ because doing so would induce posterior 0. But then a sender with $\theta = (1, 1)$ can also induce posterior μ^{L-1}, regardless of ω.

Suppose $L = 2$. Then $\mu^{L-1} = \Pr\{\omega = 1 | (\theta, \omega) \neq ((2, 1), 1)\} \equiv \bar{\mu}$. Such an equilibrium always exists. For example, consider the following strategy profile. If $\theta = (1, 0)$, then $a = (1, 0)$ for any ω. If $\theta = (2, 1)$ and $\omega = 0$, or $\theta = (1, 1)$, then the sender mixes over $a = (1, 0)$ and $a = (1, 1)$ with probability such that both arguments induce posterior $\bar{\mu}$. Note that the average posterior induced by $a = (1, 0)$ and $a = (1, 1)$ must equal $\bar{\mu}$. Therefore, such a mixture is always possible by the Intermediate Value Theorem, since $a = (1, 1)$ induces posterior q if $\theta = (1, 1)$ always makes argument $a = (1, 1)$ and $\theta = (2, 1)$ never does, and equals 0 if $\theta = (1, 1)$ always makes argument $a = (1, 0)$.

Now, suppose that $L \geqslant 3$. We claim that $\mu^{L-2} = \mu^1 = 0$, so this equilibrium induces the same distribution over posteriors as an earnest equilibrium. First, $a = (m, 0)$ cannot induce μ^{L-1} for any $m \in \{0, 1\}$, since then no sender would choose an a that induces μ^{L-2}. So μ^{L-1} must be induced by $a = (1, 1)$, $a = (0, 1)$, or both.

If $\mu^{L-2} > 0$, then any a that induces μ^{L-2} must be sent with positive probability if $\theta = (1, 0)$ and $\omega = 1$. So $a = (1, 1)$ cannot induce μ^{L-2}, since a sender with $\theta = (1, 0)$

and $\omega = 0$ cannot choose $a = (1,1)$, no other θ induces posterior μ^{L-2}, and $\mu^{L-2} < 1$. So $a = (0,1)$ must be the only argument that induces μ^{L-2}. Since $a = (0,2)$ is never sent on the equilibrium path, some $a = (m,1)$ must induce μ^{L-1}. Then a sender with type $\theta = (1,0)$ induces posterior μ^{L-1} when either $\omega = 1$, in which case the mapping from (ω, θ) to posterior beliefs is identical to an earnest equilibrium, or $\omega = 0$, in which case μ^{L-2} is induced only if $\omega = 1$ and hence $\mu^{L-2} = 1$, a contradiction. This argument proves the claim. $\blacksquare$