

1 Careers in Organizations

As we have seen in the past few weeks, treating the firm as a “black box” has simplistic implications for firm behavior and for the supply side of the economy as a whole. This treatment further has simplistic implications (and in some empirically relevant dimensions, essentially no implications) for the labor side of the economy and in particular, for workers’ careers. In an anonymous spot market for labor, individual workers have upward-sloping labor-supply curves, individual firms have downward-sloping labor-demand curves, and equilibrium wages ensure that the total amount of labor supplied in a given period is equal to the total amount of labor demanded in that period. Workers are indifferent among potential employers at the equilibrium wage, so the approach is silent on worker–firm attachment. Workers’ wages are determined by the intersection of labor supply and labor demand, so the approach predicts that variation over time in a worker’s wage is driven by aggregate changes in labor supply or labor demand. And further, the approach is agnostic about what exactly the workers do for their employers, so this approach cannot capture notions such as job assignment and promotions.

In this note, I will introduce some natural modeling elements that enrich both the labor-demand and labor-supply sides of the equation to generate predictions about the dynamics of workers’ careers, job assignments, and wages.

2 Internal Labor Markets

Doeringer and Piore (1971) define an **internal labor market** as an administrative unit “within which the pricing and allocation of labor is governed by a set of administrative rules

and procedures” rather than being determined solely by market forces. Several empirical studies using firms’ personnel data (with Baker, Gibbs, and Holmstrom (1994ab) being the focal study) highlight a number of facts regarding the operation of internal labor markets that would not arise in an anonymous spot market for labor. These facts include:

1. Many workers begin employment at the firm at a small number of positions. Doeringer and Piore refer to such positions as **ports of entry**.
2. Long-term employment relationships are common.
3. Nominal wage decreases and demotions are rare (but real wage decreases are not).
4. Workers who are promoted early on in their tenure at a firm are likely to be promoted to the next level quicker than others who were not initially promoted quickly. That is, promotions tend to be serially correlated.
5. Wage increases are serially correlated.
6. Promotions tend to be associated with large wage increases, but these wage differences are small relative to the average wage differences across levels within the firm.
7. Large wage increases early on in a worker’s tenure predict promotions.
8. There is a positive relationship between seniority and wages but no relationship between seniority and job performance or wages and contemporaneous job performance.

In addition, there are many other facts regarding the use of particular and peculiar personnel policies. For example, prior to the 1980s in the U.S., many firms made use of mandatory-retirement policies in which workers beyond a certain age were required to retire, and the firms were required to dismiss these workers. Another common policy is the use of up-or-out promotion policies, of which academics are all-too-aware. All of this is to say that the personnel policies that firms put in place are much richer and much more systematic than would be expected in an anonymous spot market for labor, and several of these facts

are consistent with workers' careers being managed at the firm-, rather than the individual-worker-, level through firm-wide policies.

There is a large and interesting theoretical literature proposing enrichments of the labor-demand or labor-supply side that in isolation generate predictions consistent with several (but typically not all) of the above features. In this note, I will focus on only a couple of the models from this literature. The models I focus on are not representative, though they do highlight a number of economic forces that are both natural and common in the literature.

2.1 Job Assignment and Human Capital Acquisition

The model in this section is based on Gibbons and Waldman (1999), and it introduces a number of important ingredients into an otherwise-standard model in order to capture many of the facts described above. First, in order for the notion of a “promotion” to be well-defined, it has to be the case that the firm has multiple jobs and reasons for assigning different workers to different jobs. In the two models I will describe here, workers in different jobs perform different activities (though in other models, such as Malcomson's (1984), this is not the case). Moreover, the models introduce heterogeneity among workers (i.e., worker “ability”) and human-capital acquisition. The two models differ in (1) how firms other than the worker's current employer draw inferences about the worker's ability and (2) the nature of human-capital acquisition.

Description There are two risk-neutral firms, F_0 and F_1 , a single risk-neutral agent A , and two periods of production. The worker's ability $\theta \in \{\theta_L, \theta_H\}$, with $\theta_L < \theta_H$ and $\Pr[\theta = \theta_H] = p$, and his work experience ℓ determine his **effective ability in period t** , $\eta_t = \theta f(\ell)$, where $f(\ell) = 1 + g\ell$, $g > 0$, and $\ell = 0$ in the first period of production and $\ell = 1$ in the second period of production. In each period, the agent can perform one of two activities for the firm that employs him. Activity 0 produces output $q^0 = d^0 + b^0(\eta_t + \varepsilon_t)$ and activity 1 produces output $q^1 = d^1 + b^1(\eta_t + \varepsilon_t)$, where $d^0 > d^1 > 0$ and $0 < b^0 < b^1$, so

that output in activity 1 is more sensitive to a worker's effective ability η and mean 0 random noise ε_t than is output in activity 0. Denote the agent's activity assignment in period t by $j_t \in \{0, 1\}$. Output in period t is therefore $q_t = (1 - j_t)q^0 + j_tq^1$. The agent's ability is symmetrically unknown, and at the end of the first period of production, both firms observe a signal $\varphi_1 \in \Phi_1 = \{q_1, j_1\}$ from which they draw an inference about η . I further assume that at the beginning of the first period of production, both firms observe $\varphi_0 \in \Phi_0 \subset \{\eta\}$. This formulation allows for the complete-information case (if $\varphi_0 = \eta$), which I will use as a benchmark. The worker's utility is

$$u_A = w_1 + w_2,$$

where w_t is his period- t wage. Firm F_i 's profits in period t are

$$\pi_{it} = q_t - w_t$$

if the agent works for F_i and 0 otherwise.

Timing The timing of the model is as follows.

1. $\theta \in \{\theta_L, \theta_H\}$ is drawn and is unobserved. φ_0 is publicly observed.
2. F_0 and F_1 simultaneously offer wages w_1^0, w_1^1 to A .
3. A chooses $d_1 \in \{0, 1\}$, where d_1 is the identity of his first-period employer, and he receives wage $w_1^{d_1}$ from F_{d_1} . Without loss of generality, assume $d_1 = 1$ (or else we can just relabel the firms).
4. F_{d_1} chooses an activity assignment $j_1 \in \{0, 1\}$, output q_1 is realized and accrues to F_1 , and both firms observe the public signal φ_1 .
5. F_0 and F_1 simultaneously offer wages w_2^0, w_2^1 to A .

6. A chooses $d_2 \in \{0, 1\}$, where d_2 is the identity of his second-period employer, and he receives wage $w_2^{d_2}$ from F_{d_2} . Assume that if A is indifferent, he chooses $d_2 = 1$.

7. F_{d_2} chooses an activity assignment $j_2 \in \{0, 1\}$. Output q_2 accrues to F_{d_2} .

Solution Concept A subgame-perfect equilibrium is a set of first-period wage offers $w_1^{0*} : \Phi_0 \rightarrow \mathbb{R}, w_1^{1*} : \Phi_0 \rightarrow \mathbb{R}$, a first-period acceptance decision rule $d_1^* : \mathbb{R}^2 \rightarrow \{0, 1\}$, a first-period job-assignment rule $j_1^{d_1^*} : \mathbb{R}^2 \times \{0, 1\} \rightarrow \{0, 1\}$, second-period wage offers $w_2^{0*} : \Phi_0 \times \mathbb{R}^2 \times \{0, 1\} \times \Phi_1 \rightarrow \mathbb{R}$ and $w_2^{1*} : \Phi_0 \times \mathbb{R}^2 \times \{0, 1\} \times \Phi_1 \rightarrow \mathbb{R}$, a second-period acceptance decision $d_2^* : \mathbb{R}^2 \times \{0, 1\} \times \mathbb{R}^2 \rightarrow \{0, 1\}$, and a second-period job assignment rule $j_2^{d_2^*} : \Phi_0 \times \mathbb{R}^2 \times \{0, 1\} \times \Phi_1 \times \mathbb{R}^2 \times \{0, 1\} \rightarrow \{0, 1\}$ such that each player's decision is sequentially optimal. The agent is said to be **promoted** if $j_2^{d_2^*} > j_1^{d_1^*}$, and he is said to be **demoted** if $j_2^{d_2^*} < j_1^{d_1^*}$.

Analysis In the second period, the agent optimally chooses to work for whichever firm offers him a higher second-period wage w_2 . In fact, both firms will offer the agent the same wage, so the agent will work for F_1 in the second period. This second-period wage will depend on the expected output the agent would produce for F_0 , given that F_0 infers something about the agent's ability θ from the public signal $\varphi = (\varphi_0, \varphi_1)$. Define the quantity $\eta_2^e(\varphi) = E[\eta_2 | \varphi]$

$$\begin{aligned} w_2^*(\varphi) &= E[(1 - j_2^{0*})q^0 + j_2^{0*}q^1 | \varphi] \\ &= (1 - j_2^{0*})(d^0 + b^0\eta_2^e(\varphi)) + j_2^{0*}(d^1 + b^1\eta_2^e(\varphi)). \end{aligned}$$

In any subgame-perfect equilibrium, both firms will choose $w_2^{i*} = w_2^*(\varphi)$. To see why, suppose the second-period wage vector $(w_2^0, w_2^1) \neq (w_2^*(\varphi), w_2^*(\varphi))$ is an equilibrium. If $w_2^i < w_2^*(\varphi)$, F_{-i} will optimally choose some $w_2^{-i} \in (w_2^i, w_2^*(\varphi))$, so F_i can always profitably deviate to some $w \in (w_2^{-i}, w_2^*(\varphi))$. If $w_2^i > w_2^*(\varphi)$, F_i can profitably deviate to $w = w_2^*(\varphi)$.

Given that both firms will choose the same wage in the second period, given the public

signal φ , the agent will work for F_1 in the second period. He will be assigned to activity 1 if

$$d^1 + b^1 \eta_2^e(\varphi) \geq d^0 + b^0 \eta_2^e(\varphi)$$

or if his expected ability is sufficiently high

$$\eta_2^e(\varphi) \geq \bar{\eta}^e \equiv \frac{d^0 - d^1}{b^1 - b^0} > 0,$$

and he will be assigned to activity 0 otherwise. Figure 1 plots $E[q^0]$ and $E[q^1]$ as a function of η^e and depicts why this activity assignment rule is optimal.

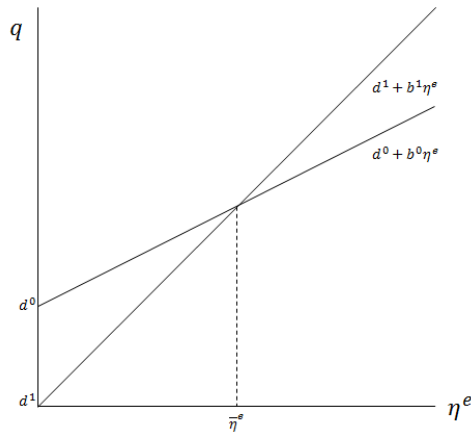


Figure 1

The first period of production is similar to the second. The agent optimally chooses to work for whichever firm offers him a higher first-period wage w_1 , and indeed both firms will offer him the same wage, so without loss of generality, we assume he works for F_1 . Again, his first-period wage depends on the expected output he would produce for F_0 given firms'

prior knowledge about θ . Define $\eta_1^e(\varphi_0) = E[\eta_1 | \varphi_0]$. His first-period wage is given by

$$\begin{aligned} w_1^* &= E[(1 - j_1^{0*})q^0 + j_1^{0*}q^1] \\ &= (1 - j_1^{0*})(d^0 + b^0\eta_1^e(\varphi_0)) + j_1^{0*}(d^1 + b^1\eta_1^e(\varphi_0)), \end{aligned}$$

and again, his first-period employer will optimally assign him to activity 1 if and only if

$$\eta_1^e(\varphi_0) \geq \bar{\eta}^e = \frac{d^0 - d^1}{b^1 - b^0}.$$

Importantly, the threshold is the same in each period, even though $E[\eta_2^e(\varphi)] = (1 + g)\eta_1^e(\varphi_0) > \eta_1^e(\varphi_0)$.

Discussion Slight extensions of this model generate a number of predictions that are consistent with several of the facts I outlined in the discussion above. First, if p is sufficiently low, then all workers begin their employment spell by performing activity 1, which therefore serves as a port of entry into the firm. Long-term employment relationships are common, although this result follows because of the particular tie-breaking rule I have assumed—as we will see in the next model, if human capital acquisition is firm-specific rather than general, long-term employment relationships would arise for other tie-breaking rules as well.

Next, demotions are rare in this model. To see why, suppose that it is optimal to assign the agent to activity 1 in the first period. That is,

$$\hat{p}(\varphi_0)\theta^H + (1 - \hat{p}(\varphi_0))\theta^L \geq \frac{d^0 - d^1}{b^1 - b^0},$$

where $\hat{p}(\varphi_0)$ is the conditional probability that $\theta = \theta^H$ given public signal φ_0 . In order for the agent to be demoted in period 2, if we denote by $\hat{p}(\varphi)$ the conditional probability that

$\theta = \theta^H$ given public signal φ , it must be the case that

$$\hat{p}(\varphi)\theta^H + (1 - \hat{p}(\varphi))\theta^L \geq \frac{1}{1+g} \frac{d^0 - d^1}{b^1 - b^0}.$$

If $\varphi_0 = \eta$, so that we are in a complete-information environment, then workers are never demoted, because $\hat{p}(\varphi) = \hat{p}(\varphi_0)$. If $\varphi_0 = \emptyset$, so that we are in a symmetric-learning environment, then demotions are rare, because $E[\hat{p}(\varphi)] = p$, so that in expectation the left-hand side of the second-period cutoff is the same as the left-hand side of the first-period cutoff, but the right-hand side is strictly smaller. Wage cuts are also rare for the same reason.

The model also generates the prediction that promotions tend to be associated with especially large wage increases. This is true for both the complete-information and the symmetric-learning versions of the model. In the complete-information model, the wage increase for a worker conditional on not being promoted (i.e., if the parameters were such that the worker is optimally assigned to activity 0 in both periods) is $\theta b^0 g$ (since $w_2 = d^0 + b^0 \theta (1 + g)$ and $w_1 = d^0 + b^0 \theta$). Analogously, the wage increase for a worker conditional on being promoted is $d^1 - d^0 + \theta (b^1 - b^0) + \theta b^1 g$. Since the worker is optimally being promoted, it has to be the case that $d^1 - d^0 + \theta (b^1 - b^0) > 0$, so this wage increase exceeds $\theta b^1 g$, which is certainly higher than $\theta b^0 g$ conditional on θ . Moreover, for it to be optimal to promote some workers but not all workers, it must be the case that the promoted workers have $\theta = \theta^H$, and the workers who are not promoted have $\theta = \theta^L$, further widening the difference in wage increases. This justification for wage jumps at promotion is a bit unsatisfying, and this is an issue that the model in the next section is partly designed to address.

With only two periods of production and two activities, it is not possible for the model to deliver serially correlated wage increases and promotions, but with more periods and more activities, it is.

2.2 Promotions as Signals

Description There are two firms, F_0 and F_1 , a single agent A , and two periods of production. In each period, the agent can perform one of two activities for the firm that employs him. Activity 0 produces output that is independent of the agent's ability θ , and activity 1 produces output that is increasing in his ability. Output is sold into a competitive product market at price 1. The agent's ability is $\theta \sim U[0, 1]$, and it is symmetrically unknown at the beginning of the game, but it is observed at the end of the first period of production by the agent's first-period employer but not by the other firm. The other firm will infer something about the worker's ability by his first-period employer's decision about his second-period activity assignment: promotions will therefore serve as a signal to the market. The agent acquires firm-specific human capital for his first-period employer.

In the first period, the worker produces an amount $q_1 = x \in (1/2, 1)$ for his employer if he is assigned to activity 0 and $q_1 = \theta$ if he is assigned to activity 1, so that his first-period employer will always assign him to activity 0, since $E[\theta] = 1/2 < x$. In the second period, if he is assigned to activity j , he produces $q_2(j, \theta, d_2) = (1 + s1_{d_2=d_1})[(1 - j)x + j\theta]$, where $1_{d_2=d_1}$ is an indicator variable for the event that the worker works for the same firm in both periods and $s \geq 0$ represents firm-specific human capital. The worker's utility is

$$u_A = w_1 + w_2,$$

where w_t is his period- t wage. Firm F_i 's profits in period t are

$$\pi_{it} = q_t - w_t$$

if the agent works for F_i and 0 otherwise.

Timing The timing of the model is as follows.

1. F_0 and F_1 simultaneously offer wages w_1^0, w_1^1 to A .

2. A chooses $d_1 \in \{0, 1\}$, where d_1 is the identity of his first-period employer, and he receives wage $w_1^{d_1}$ from F_{d_1} . Without loss of generality, assume $d_1 = 1$ (or else we can just relabel the firms).
3. $\theta \sim U[0, 1]$ is drawn. θ is observed by F_1 . Output q_1 is realized and accrues to F_1 .
4. F_1 offers A a pair (j^1, w_2^1) consisting of a second-period activity assignment $j^1 \in \{0, 1\}$ and a second-period wage. j^1 is commonly observed, but w_2^1 is not observed by F_0 .
5. F_0 offers A a pair (j^0, w_2^0) . This offer is observed by A .
6. A chooses $d_2 \in \{0, 1\}$, where d_2 is the identity of his second-period employer, and he receives wage $w_2^{d_2}$ from F_{d_2} . Assume that if A is indifferent, he chooses $d_2 = 1$.
7. Output $q_2(j, \theta, d_2)$ accrues to F_{d_2} .

Solution Concept A Perfect-Bayesian equilibrium (PBE) is a belief assessment μ , first-period wage offers $w_1^{0*}, w_1^{1*} \in \mathbb{R}$, a first-period acceptance decision rule $d_1^* : \mathbb{R}^2 \rightarrow \{0, 1\}$, a second-period job assignment $j^{1*} : \mathbb{R}^2 \times \{0, 1\} \times [0, 1] \rightarrow \{0, 1\}$ and wage offer $w_2^{1*} : \mathbb{R}^2 \times \{0, 1\} \times [0, 1] \rightarrow \mathbb{R}^2$ by F_1 , a second-period offer $(j^{0*}, w_2^{0*}) : \mathbb{R}^2 \times \{0, 1\} \times \{0, 1\} \rightarrow \{0, 1\} \times \mathbb{R}$ by F_0 , and a second-period acceptance decision $d_2^* : \mathbb{R}^2 \times \{0, 1\} \times \{0, 1\}^2 \times \mathbb{R}^2 \rightarrow \{0, 1\}$ such that each player's decision is sequentially optimal, and beliefs are consistent with Bayes's rule whenever possible. A **promotion rule** is a mapping from θ to $\{0, 1\}$. Firm F_1 's optimal promotion rule will turn out to be a threshold promotion rule, which greatly simplifies the analysis.

Analysis In the second period, the agent optimally chooses to work for whichever firm offers him a higher second-period wage w_2 . In fact, in every PBE, both firms will offer the agent the same wage, so the agent will work for F_1 in the second period. This second-period wage will, however, depend on the expected output the agent would produce for F_0 ,

given that F_0 infers something about θ from F_1 's second-period activity-assignment decision. Define the quantity

$$w_2^*(j^1) = E[(1 - j^{0*})x + j^{0*}\theta | j^{1*}(\theta) = j^1].$$

$w_2^*(j^1)$ is equal to the expected output of F_0 if it employs A in the second period, given F_1 's equilibrium promotion rule and its outcome j^1 .

Result 1. In any PBE in undominated strategies, $w_2^{0*} = w_2^{1*} = w_2^*(j^1)$.

Proof of Result 1. In any PBE in undominated strategies, given j^1 , we must have that $w_2^{0*} \leq w_2^*(j^1)$. Suppose $w_2^{1*} < w_2^*(j^1)$. Then F_0 will optimally choose some $w_2^0 \in (w_2^{1*}, w_2^*(j^1))$, and F_1 would earn zero profits, but F_1 could guarantee itself strictly positive profits by deviating to $w_2^1 = w_2^*(j^1)$, because A would then choose $d_2 = 1$, and $w_2^*(j^1)$ is strictly less than F_1 's expected output in period 2, because of firm-specific human capital. If $w_2^{1*} > w_2^*(j^1)$, then F_1 could increase its profits by deviating to any $w_2^1 \in (w_2^*(j^1), w_2^{1*})$, since F_0 offers at most $w_2^*(j^1)$, and therefore A will still choose $d_2 = 1$. It must therefore be the case that $w_2^{1*} = w_2^*(j^1)$. A best response of F_0 to $w_2^{1*} = w_2^*(j^1)$ is to choose $w_2^{0*} = w_2^*(j^1)$. ■

Firm F_1 has to choose between “promoting” the Agent and offering him $(1, w_2^*(1))$ and not promoting him, offering $(0, w_2^*(0))$. F_1 therefore chooses $j^{1*}(\theta)$ to solve

$$\max_{j^1} \underbrace{\{(1 + s)[(1 - j^1)x + j^1\theta] - w_2^*(j^1)\}}_{\pi_1(j^1, \theta)}.$$

The function $\pi_1(j^1, \theta)$ has increasing differences in j^1 and θ , so F_1 's optimal promotion rule will necessarily be monotone increasing in θ , and therefore $j^{1*}(\theta)$ will be a threshold promotion rule.

Result 2. In any PBE in undominated strategies, F_1 chooses a threshold promotion rule

$$j^{1*}(\theta) = \begin{cases} 0 & 0 \leq \theta < \hat{\theta} \\ 1 & \hat{\theta} \leq \theta \leq 1, \end{cases}$$

for some threshold $\hat{\theta}$.

It therefore remains to determine the equilibrium threshold $\hat{\theta}^*$. Given a threshold $\hat{\theta} \in (0, 1)$, the expected ability of promoted workers is $E[\theta | j^1 = 1] = (1 + \hat{\theta})/2$, and the expected ability of non-promoted workers is $E[\theta | j^1 = 0] = \hat{\theta}/2$. The wages for promoted workers and non-promoted workers are therefore

$$\begin{aligned} w_2^*(j^1 = 1; \hat{\theta}) &= \max \left\{ x, \frac{1 + \hat{\theta}}{2} \right\} \\ w_2^*(j^1 = 0; \hat{\theta}) &= \max \left\{ x, \frac{\hat{\theta}}{2} \right\} = x, \end{aligned}$$

where the last equality holds, because $x > 1/2$, and therefore $x > \hat{\theta}/2$. Given these wage levels as a function of the equilibrium threshold $\hat{\theta}^*$, the equilibrium threshold $\hat{\theta}^*$ is the θ that makes F_1 indifferent between promoting the Agent and not:

$$(1 + s)x - x = (1 + s)\hat{\theta}^* - \max \left\{ x, \frac{1 + \hat{\theta}^*}{2} \right\}.$$

The equilibrium threshold $\hat{\theta}^*$ necessarily satisfies $(1 + \hat{\theta}^*)/2 > x$. If this were not the case, then the indifference condition above would imply that $\hat{\theta}^* = x$, which would in turn contradict the presumption that $(1 + \hat{\theta}^*)/2 < x$, since $x < 1$. The indifference condition therefore uniquely pins down the equilibrium threshold $\hat{\theta}^*$.

Result 3. In any PBE in undominated strategies, the promotion threshold is given by

$$\hat{\theta}^* = \frac{1 + 2sx}{1 + 2s},$$

which is strictly greater than x and weakly less than one.

We can now contrast the equilibrium promotion rule to the first-best promotion rule. Under a first-best promotion rule, the Agent would be assigned to activity 1 whenever $\theta \geq x$. In contrast, in any PBE in undominated strategies, the Agent is assigned to activity 1 whenever $\theta \geq \hat{\theta}$, where $\hat{\theta} > x$. That is, the firm fails to promote the agent when it would be socially efficient to do so. Indeed, when $s = 0$, the firm promotes the agent with probability zero. When the firm promotes the worker, his outside option increases, because a promotion is a positive signal about his ability, and so the firm has to raise his wage in order to prevent him from going to the other firm. Promoting the worker increases the firm's output by $(1 + s)(\theta - x)$, but it also increases the worker's wage by $\frac{1 + \hat{\theta}^*}{2} - x$, which is equal to $(1 + s)\frac{1 - x}{1 + 2s}$.

Further Reading The theoretical literature on the reasons for and properties of internal labor markets is large. Waldman (1984), Bernhardt (1995), Ghosh and Waldman (2010), Bose and Lang (2013), and Bond (2015) provide symmetric- and asymmetric-learning-based models. Lazear and Rosen (1981), Malcomson (1984), Rosen (1986), MacLeod and Malcomson (1988), Milgrom and Roberts (1988), Prendergast (1993), Chan (1996), Zabochnik and Bernhardt (2001), Waldman (2003), Krakel and Schottner (2012), Auriol, Friebel, and von Bieberstein (2016), and Ke, Li, and Powell (2017, 2018) provide incentives-based models. Prendergast (1993), Demougin and Siow (1994), Zabochnik and Bernhardt (2001), Camara and Bernhardt (2009), and DeVaro and Morita (2013) provide models based on human-capital acquisition.