Economics 2010b: Microeconomic Theory II

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Financial Contracting

1 Financial Contracting

The last topic that we will cover in this class applies the tools we have developed over the last couple weeks in order to think about corporate governance, which Shleifer and Vishny (1997) define as "ways in which the suppliers of finance to corporations assure themselves of getting a return on their investment." We will think about a setting in which a capital-constrained Entrepreneur needs capital from capital-rich potential Investor to undertake a project that yields a positive return. We will look at the different instruments the Entrepreneur has to credibly commit herself to return funds to such an Investor in order to attract financing from them.

In a world of complete contracts and complete financial markets, how a project is financed—whether through debt or equity or some other, more complicated arrangement—is irrelevant for the total value of the project, and every positive net-present value project will be funded. The irrelevance result is known as the Modigliani-Miller theorem (Modigliani and Miller, 1958) and it is not so different from versions of the Coase theorem that we have mentioned in passing a few times. (Very) roughly speaking, we can think of the expected discounted revenues from the project as some value V. If undertaking the project requires K dollars worth of capital, then the Investor has to get at least K dollars back. One way he could get K dollars back is if he gets a share of the future revenues for which the expected present discounted value is K. Or the Entrepreneur could write a debt contract for which the expected present discounted value of payments is K. Either way, the Entrepreneur will receive $V - K$ and will undertake the project if $V > K$.

The Modigliani-Miller theorem served as a benchmark and spawned a literature providing explanations for when and why debt has advantages over equity based on two classes of explanations: differences in tax treatment and incentive problems. Our focus will be on the latter and in particular on how different arrangements lead the Entrepreneur to make different decisions that in turn affect the value of the project. Without appropriate contractual safeguards, the Investor might worry that the Entrepreneur will make decisions that are privately beneficial to the Entrepreneur but harmful to the Investor. The moral hazard problems that arise in these settings may include insufficient effort on the part of the Entrepreneur, although this may not take the form of the Entrepreneur working too few hours, but rather that she might avoid unpleasant tasks like firing people or a taking a tough stance in negotiations with suppliers. The problem may take the form of unnecessary or extravagant investments aimed at growing the Entrepreneur's "empire" at the expense of the Investor's returns. Or it may take the form of self dealing and excessive perk consumption: buying costly private jets, expensive art for the corporate headquarters, or hiring friends and family members.

When actions like the ones described above are not contractible, credit may be *rationed* in the sense that the Entrepreneur may be unable to "obtain the loan [she] wants even though [she] is willing to pay the interest that the lenders are asking, and perhaps even a higher interest rate." (Tirole, 2005, p. 113) Positive net-present value projects may therefore not be undertaken. We will begin with a workhorse model that builds off our analysis of limited liability constraints to provide a reason why credit may be rationed. As in our earlier discussion of such models, the Entrepreneur must be given a rent in order to provide her with incentives to take the right action. The total returns from the Entrepreneur's project net of the incentive rents the Entrepreneur must receive is what we will refer to as her *pledgeable income*. Even if the overall income from the project would be high enough to cover the Investor's capital costs, if the Entrepreneur's pledgeable income is not, she will be unable to attract funding from the Investor.

The form of the optimal contract in this model can, depending on how you look at it, be interpreted either as a debt contract or as a contract involving outside equity. But it lacks the richness of form that real financing arrangements take. In particular, when we think of equity, we typically think of a contract in which an outside Investor owns some share of a firm's profits and is also able to exercise some limited control over some of the firm's decisions. When we think of debt, we think of contracts in which the Investor is guaranteed some payments, and if the Entrepreneur does not repay the Investor, the Investor gains control over the associated assets and can then make decisions about how they are used. The model above has no notion of control rights, so it is unable to provide a compelling argument for why such contracts might move around control rights in a contingent way. We will therefore take an incomplete contracts view to think about how contingent control rights might be used in an optimal arrangement.

2 Pledgeable Income and Credit Rationing

There is a risk-neutral Entrepreneur (E) and a risk-neutral Investor (I). The Investor has capital but no project, and the Entrepreneur has a project but no capital. In order to pursue the project, the Entrepreneur needs K units of capital. Once the project has been pursued, the project yields revenues py, where $y \in \{0, 1\}$ is the project's output, and p is the market price for that output. The Entrepreneur chooses an action $e \in [0, 1]$ that determines the probability of a successful project, $\Pr[y = 1 | e] = e$, as well as a private benefit $b(e)$ that accrues to the Entrepreneur, where b is strictly decreasing and concave in e and satisfies $b'(0) = 0$ and $\lim_{e \to 1} b'(e) = -\infty$.

The Entrepreneur can write a contract $w \in \mathcal{W} = \{w : \{0, 1\} \to \mathbb{R},\ 0 \leq w(y) \leq py\}$ that pays the Investor $w(y)$ if output is y and therefore shares the projects revenues with the Investor. If the Investor declines the contract, he keeps the K units of capital, and the Entrepreneur receives a payoff of 0. If the Investor accepts the contract, the Entrepreneur's and Investor's preferences are

$$\begin{aligned} U_E(w,e) &= E[py - w(y)|\, e] + b(e) \\ U_I(w,e) &= E[w(y)|\, e]. \end{aligned}$$

There are strong parallels between this model and the limited-liability Principal-Agent model we studied earlier. We can think of the Entrepreneur as the Agent and the Investor as the Principal. There is one substantive difference and two cosmetic differences. The substantive difference is that the Entrepreneur is the one writing the contract, and while the contract must still satisfy the Entrepreneur's incentive-compatibility constraint, the individual rationality constraint it has to satisfy is the *Investor's*. The two cosmetic differences are: (1) the payments in the contract flow from the Entrepreneur to the Investor, and (2) instead of higher values of e costing the Entrepreneur $c(e)$, they reduce her private benefits $b(e)$.

Timing The timing of the game is as follows.

1. E offers I a contract $w(y)$, which is commonly observed.
2. I accepts the contract $(d = 1)$ or rejects it $(d = 0)$ and keeps K, and the game ends. This decision is commonly observed.
3. If I accepts the contract, E chooses action e and receives private benefit $b(e)$. e is only observed by E.
4. Output $y \in \{0,1\}$ is drawn, with $\Pr[y = 1|\, e] = e$. y is commonly observed.
5. E pays I an amount $w(y)$. This payment is commonly observed.

Equilibrium The solution concept is the same as always. A **pure-strategy subgame-perfect equilibrium** is a contract $w^* \in \mathcal{W}$, an acceptance decision $d^* : \mathcal{W} \to \{0,1\}$, an action choice $e^* : \mathcal{W} \times \{0,1\} \to [0,1]$ such that given contract w^*, the Investor optimally chooses d^*, and the Entrepreneur optimally chooses e^*, and given d^*, the Investor optimally offers contract w^*. We will say that the optimal contract induces action e^*.

The Program The Entrepreneur offers a contract $w \in \mathcal{W}$, which specifies a payment $w(0) = 0$ and $0 \leq w(1) \leq p$ and proposes an action e to solve

$$\max_{w(1),e} (p - w(1))\, e + b(e)$$

subject to the incentive-compatibility constraint

$$e \in \underset{\hat{e}\in[0,1]}{\operatorname{argmax}} (p - w(1))\, \hat{e} + b(\hat{e}),$$

the Investor's individual-rationality (or break-even) constraint

$$w(1)\, e \geq K.$$

Analysis We can decompose the problem into two steps. First, we can ask: for a given action e, how much rents must the Entrepreneur receive in order to choose action e, and therefore, what is the maximum amount that the Investor can be promised if the Entrepreneur chooses e? Second, we can ask: given that the Investor must receive K, what action e^* maximizes the Entrepreneur's expected payoff?

The following figure illustrates the problem using a graph similar to the one we looked at when we thought about limited liability constraints. The horizontal axis is the Entrepreneur's action e, and the segment pe is the expected revenues as a function of e. The dashed line $(p - w_{e_1})\, e$ represents, for a contract that pays the Investor $w(1) = w_{e_1}$ if $y = 1$, the Entrepreneur's expected monetary payoff, and $-b(e)$ represents the Entrepreneur's cost of choosing different actions. As the figure illustrates, the contract that gets the Entrepreneur to choose action e_1 can pay the Investor at most $w_{e_1} e_1$ in expectation.

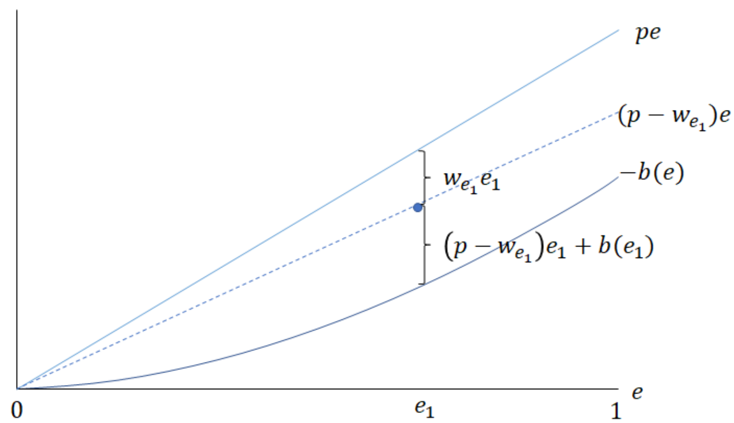

Figure 9: Entrepreneur Incentive Rents

The next figure illustrates, for different actions e, the rents $(p - w_e)\,e + b\,(e)$ that the Entrepreneur must receive for e to be incentive-compatible. Note that because $w_e \geq 0$, there is no incentive-compatible contract that gets the Entrepreneur to choose any action $e > e^{FB}$. The vertical distance between the expected revenue pe curve and the Entrepreneur rents curve is the Investor's expected payoff under the contract that gets the Entrepreneur to choose action e. For the Investor to be willing to sign such a contract, that vertical distance must be at least K, which is the amount of capital the Entrepreneur needs.

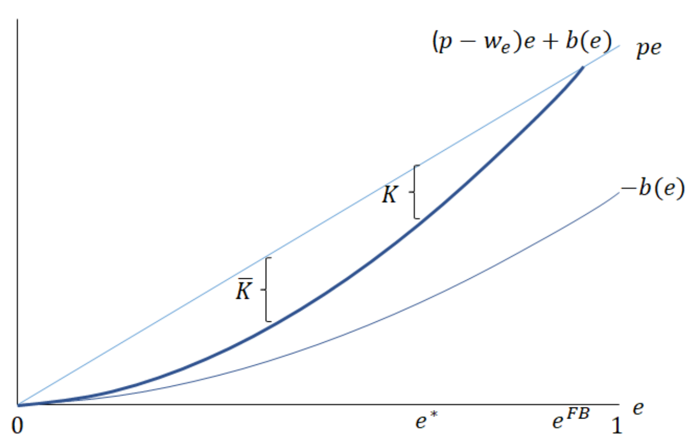

Figure 10: Equilibrium and Pledgeable Income

Two results emerge from this analysis. First, if $K > 0$, then in order to secure funding K, the Entrepreneur must share some of the project's earnings with the Investor, which means that the Entrepreneur does not receive all the returns from her actions and therefore will choose an action $e^* < e^{FB}$. Second, the value $\bar{K}$ represents the maximum expected payments the Entrepreneur can promise the Investor in any incentive-compatible contract. This value is referred to as the Entrepreneur's **pledgeable income**. If the project requires capital $K > \bar{K}$, then there is no contract the Entrepreneur can offer the Investor that the Investor will be willing to sign, even though the Entrepreneur would invest in the project if she had her own capital. When this is the case, we say that there is **credit rationing**.

As a final point about this model, with binary output, the optimal contract can be interpreted as either a debt contract or an equity contract. Under the debt contract interpretation, the Entrepreneur must reimburse w_{e^*} or else go bankrupt, and if the project is successful, she keeps the residual $p - w_{e^*}$. Under the equity contract interpretation, the Entrepreneur holds a share $(p - w_{e^*})/p$ of the project's equity, and the Investor holds a share w_{e^*}/p of the project's equity. That the optimal contract can be interpreted as either a debt contract or an equity contract highlights that if we want to actually understand the role of debt or equity contracts, we will need a richer model.

3 Control Rights and Financial Contracting

The previous model cannot explain the fact that equity has voting power while debt does not, except following default. Aghion and Bolton (1992) takes an incomplete contracting approach to thinking about financial contracting and brings control rights front and center. We will look at a simple version of the model that provides an explanation for debt contracts featuring *contingent* control. In this model, control rights matter because the parties disagree about important decisions that are ex ante noncontractible. The parties will renegotiate over these decisions ex post, but because the Entrepreneur is wealth-constrained, renegotiation may not fully resolve the disagreement. Investor control will therefore lead to a smaller pie ex post, but the Investor will receive a larger share of that pie. As a result, even though Investor control destroys value, it may be the only way to get the Investor to be willing to invest to begin with.

The Model As in the previous model, there is a risk-neutral Entrepreneur (E) and a risk-neutral investor (I). The Investor has capital but no project, the Entrepreneur has a project but no capital, and the project costs K. The parties enter into an agreement, which specifies who will possess the right to make a decision $d \in \mathbb{R}_+$ once that decision needs to be made. After the state $\theta \in \mathbb{R}_+$, which is drawn according to density $f(\theta)$, is realized, the decision d is made. This decision determines verifiable profits $y(d)$, which we will assume accrue to the Investor.[1] It also determines nonverifiable private benefits $b(d)$ that accrue to the Entrepreneur.

The parties can contract upon a rule that specifies who will get to make the decision d in which state of the world: let $g : \mathbb{R}_+ \to \{E, I\}$ denote the **governance structure**, where $g(\theta) \in \{E, I\}$ says who gets to make the decision d in state θ. The decision d is itself not ex ante contractible, but it is ex post contractible, so that the parties can negotiate over it ex post. In particular, we will assume that the Entrepreneur has all the bargaining power, so that she will propose a take-it-or-leave-it offer specifying a decision d as well as a transfer $w \geq 0$ from the Investor to the Entrepreneur. Note that the transfer has to be nonnegative, because the Entrepreneur is cash-constrained.

Timing

1. E proposes a governance structure g. g is commonly observed.

[1] We could enrich the model to allow the parties to contract ex ante on the split of the verifiable profits that each party receives. Giving all the verifiable profits to the Investor maximizes the efficiency of the project because it maximizes the pledgeable income that he can receive without having to distort ex post decision making.

2. I chooses whether or not to go ahead with the investment. This decision is commonly observed.

3. The state θ is realized and is commonly observed.

4. E makes a take-it-or-leave-it offer of (d, w) to I, who either accepts or rejects it.

5. If I rejects the offer, party $g(\theta)$ chooses d.

Analysis As usual, let us start by describing the first-best decision that maximizes the sum of the profits and the private benefits:

$$d^{FB} \in \underset{d \in \mathbb{R}_+}{\operatorname{argmax}}\, y(d) + b(d).$$

Assume y and b are strictly concave and single-peaked, so that there is a unique first-best decision. Moreover, assume $y(d)$ is maximized at some decision d^I, and $b(d)$ is maximized at some other decision $d^E < d^I$. These assumptions imply that $d^E < d^{FB} < d^I$. Now, let us see what happens depending on who has control.

We will first look at what happens under Entrepreneur control. This corresponds to $g(\theta) = E$ for all θ. In this case, if the Investor rejects the Entrepreneur's offer in stage 4, the Entrepreneur will choose d to maximize her private benefit and will therefore choose d^E. Recall that the Entrepreneur does not care about the profits of the project because we have assumed that the profits accrue directly to the Investor. The decision d^E is therefore the Investor's outside option in stage 4. It will not be the decision that is actually made, however, because the Entrepreneur can offer to make a higher decision in exchange for some money. In particular, she will offer (d^{FB}, w), where w is chosen to extract all the ex post surplus from the Investor:

$$y\left(d^{FB}\right) - w = y\left(d^E\right) \text{ or } w = y\left(d^{FB}\right) - y\left(d^E\right) > 0.$$

Under Entrepreneur control, the Entrepreneur's payoff will therefore be $b\left(d^{FB}\right)+y\left(d^{FB}\right)-y\left(d^{E}\right)>b\left(d^{E}\right)$, and the Investor's payoff will be $y\left(d^{E}\right)$, which is effectively the Entrepreneur's pledgeable income. If $y\left(d^{E}\right)>K$, then the Investor will make the investment, and the first-best decision will be made, but if $y\left(d^{E}\right)<K$, this arrangement will not get the Investor to make the investment.

Now let us look at what happens under Investor control, which corresponds to $g(\theta)=I$ for all θ. In this case, if the Investor rejects the Entrepreneur's offer at stage 4, the Investor will choose d to maximize profits and will therefore choose d^{I}. The decision d^{I} is therefore the Investor's outside option in stage 4. At stage 4, the Entrepreneur would like to get the Inventor to make a decision $d<d^{I}$, but in order to get him to do so, she would have to choose $w<0$, which is not feasible. As a result, d^{I} will in fact be the decision that is made. Under Investor control, the Entrepreneur's payoff will be $b\left(d^{I}\right)$, and the Investor's payoff will be $y\left(d^{I}\right)$, which again is effectively the Entrepreneur's pledgeable income. Conditional on the investment being made, total surplus under Investor control is lower than under Entrepreneur control, but the benefit of Investor control is that it ensures the Investor a payoff of $y\left(d^{I}\right)$, which may exceed K even if $y\left(d^{E}\right)$ does not.

As in the Property Rights Theory, decision rights determine parties' outside options in renegotiations, which determines their incentives to make investments that are specific to the relationship. In contrast to the PRT, however, ex post renegotiation does not always lead to a surplus-maximizing outcome because the Entrepreneur is wealth-constrained. As such, in order to provide the Investor with incentives to make the relationship-specific investment of investing in the project, we may have to give the Investor ex post control, even though he will use it in a way that destroys total surplus.

If $y\left(d^{I}\right)>K>y\left(d^{E}\right)$, then Investor control is better than Entrepreneur control because it ensures the Investor will invest, but in some sense, it involves throwing away more surplus than necessary. In particular, consider a governance structure $g(\cdot)$ under which the Entrepreneur has control with probability π (i.e., $\Pr\left[g(\theta)=E\right]=\pi$), and the Investor has control with probability $1-\pi$ (i.e., $\Pr[g(\theta)=I]=1-\pi$). The Entrepreneur can get the Investor to invest if she chooses π to satisfy

$$\pi y\left(d^{E}\right)+(1-\pi)\, y\left(d^{I}\right)=K,$$

which will be optimal.

Now, stochastic control in this sense is a bit tricky to interpret, but with a slight elaboration of the model, it has a more natural interpretation. In particular, suppose that the state of the world, θ, determines how sensitive the project's profits are to the decision, so that

$$y(d,\theta)=\alpha(\theta)\, y(d)+\beta(\theta),$$

where $\alpha(\theta)>0$, and $\alpha'(\theta)<0$. In this case, the optimal governance structure would involve a cutoff θ^{*} so that $g(\theta)=E$ if $\theta>\theta^{*}$ and $g(\theta)=I$ if $\theta\leq\theta^{*}$, where this cutoff is chosen so that the Investor's expected payoffs would be K.

If $\alpha'(\theta)\, y(d)+\beta'(\theta)>0$ for all d, then high-θ states correspond to high-profit states, and this optimal arrangement looks somewhat like a debt contract that gives control to the creditor in bad states and gives control to the Entrepreneur in the good states. In this sense, the model captures an important aspect of debt contracts, namely that they involve contingent allocations of control. This theory of debt contracting is not entirely compelling, though, because the most basic feature of debt contracts is that the shift in control to the Investor occurs *only if the Entrepreneur does not make a repayment.* The last model we will look at will have this feature.

4 Cash Diversion and Liquidation

We will look at one final model that involves an important decision that is often specified in debt contracts: whether to liquidate an ongoing project. We will show that when the firm's cash flows are noncontractible, giving the Investor the rights to the proceeds from a liquidation event can protect him from short-run expropriation from an Entrepreneur who may want to direct the project's cash flows toward her own interests.

The Model As before, there is a risk-neutral Entrepreneur (E) and a risk-neutral investor (I). The Investor has capital but no project, the Entrepreneur has a project but no capital, and the project costs K. If the project is funded, it yields income over two periods, which accrue to the Entrepreneur. In the first period, it produces output $y_1 \in \mathcal{Y}_1 \equiv \{0, 1\}$, where $\Pr[y_1 = 1] = q$, and that output generates a cash flow of $p_1 y_1$. After y_1 is realized, the Entrepreneur can make a cash payment $0 \leq \hat{w}_1 \leq p_1 y_1$ to the Investor. The project can then be terminated, yielding a liquidation value of L, where $0 \leq L \leq K$, which accrues to the Investor. Denote the probability the project is continued by $r \in [0, 1]$. If the project is continued, in the second period, it produces output $y_2 = 1$, and that output generates cash flow of p_2. At this point, the Entrepreneur can again make a cash payment $0 \leq \hat{w}_2 \leq p_2$ to the Investor.

The cash flows are noncontractible, so the parties are unable to write a contract that specifies output-contingent repayments from the Entrepreneur to the Investor, but they can write a contract that specifies probabilities $r : \mathbb{R}_+ \to [0, 1]$ that determine the probability $r(\hat{w}_1)$ the project is continued if the Entrepreneur pays the Investor $\hat{w}_1$. The contracting space is therefore $\mathcal{W} = \{r : \mathbb{R}_+ \to [0, 1]\}$. The players' payoffs, if the Investor invests K in the project are:

$$
\begin{aligned}
u_E(\ell, y_1, \hat{w}_1, \hat{w}_2) &= p_1 y_1 - \hat{w}_1 + r(\hat{w}_1)(p_2 - \hat{w}_2) \\
u_I(\ell, y_1, \hat{w}_1, \hat{w}_2) &= \hat{w}_1 + (1 - r(\hat{w}_1)) L + r(\hat{w}_1) \hat{w}_2.
\end{aligned}
$$

Throughout, we will assume that $p_2 > L$, so that liquidation strictly reduces total surplus.

Timing The timing of the game is as follows.

1. E offers I a contract $r(\hat{w}_1)$, which is commonly observed.

2. I accepts the contract $(d = 1)$ or rejects it $(d = 0)$ and keeps K, and the game ends. This decision is commonly observed.

3. If I accepts the contract, output $y_1 \in \{0, 1\}$ is realized. y_1 is commonly observed.

4. E makes a payment $0 \leq \hat{w}_1 \leq p_1 y_1$ to I. $\hat{w}_1$ is commonly observed.

5. The project is liquidated with probability $1 - r(\hat{w}_1)$. The liquidation event is commonly observed.

6. If the project has not been liquidated, output $y_2 = 1$ is realized. y_2 is commonly observed.

7. E makes a payment $0 \leq \hat{w}_2 \leq y_2$ to I. $\hat{w}_2$ is commonly observed.

Equilibrium The solution concept is the same as always. A **pure-strategy subgame-perfect equilibrium** is a continuation function $r^* \in \mathcal{W}$, an acceptance decision $d^* : \mathcal{W} \to \{0, 1\}$, a first-period payment rule $w_1^* : \mathcal{W} \times \{0, 1\} \to \mathbb{R}_+$, and a second-period payment rule $w_2^* : \mathcal{W} \times \{0, 1\} \times \{0, 1\} \times \mathbb{R}_+ \to \mathbb{R}_+$ such that given continuation function r^* and payment rules w_1^* and w_2^*, the Investor optimally chooses d^*, and given d^*, the Entrepreneur optimally offers continuation function r^* and chooses payment rules w_1^* and w_2^*.

The Program Models such as this one, in which the Entrepreneur's repayment decisions are not contractible, are referred to as **cash diversion** models. The Entrepreneur's problem will be to write a contract that specifies continuation probabilities and repayment amounts so that given those repayment-contingent continuation probabilities, the Entrepreneur will actually follow through with those repayments, and the Investor will at least break even. In this setting, it is clear that in any subgame-perfect equilibrium, the Entrepreneur will not make any positive payment $\hat{w}_2 > 0$, since she receives nothing in return for doing so. Moreover, it will be without loss of generality for the Entrepreneur to specify a single repayment amount $0 < w_1 \leq p_1$ to be repaid if $y_1 = 1$, and a pair of probabilities r_0 and r_1, where r_0 is the probability the project is continued (and not liquidated) if $\hat{w}_1 \neq w_1$, and r_1 is the probability the project is continued if $\hat{w}_1 = w_1$. The Entrepreneur's problem is therefore

$$\max_{r_0, r_1, w_1 \leq p_1} q\left(p_1 - w_1 + r_1 p_2\right) + (1 - q)\, r_0 p_2$$

subject to the Entrepreneur's incentive-compatibility constraint

$$p_1 - w_1 + r_1 p_2 \geq p_1 + r_0 p_2$$

and the Investor's break-even constraint

$$q\left(w_1 + (1 - r_1)\, L\right) + (1 - q)\,(1 - r_0)\, L \geq K.$$

It will be useful to rewrite the incentive-compatibility constraint as

$$(r_1 - r_0)\, p_2 \geq w_1,$$

which says that in order for repayment w_1 to be incentive-compatible, it has to be the case that by making the payment w_1 (instead of paying zero), the probability r_1 that the project is continued (and hence the Entrepreneur receives p_2) if she makes the payment is sufficiently high relative to the probability r_0 the project is continued when she does not make the payment.

Analysis In order to avoid multiple cases, we will assume that

$$p_1 > \frac{p_2}{q p_2 + (1 - p)\, L} K,$$

which will ensure that in the optimal contract, the Entrepreneur's first-period payment will satisfy $w_1^* < p_1$.

The Entrepreneur's problem is just a constrained maximization problem with a linear objective function and linear constraints, so it can in principle be easily solved using standard linear-programming techniques. We will instead solve the problem by thinking about a few perturbations that, at the optimum, must not be profitable. Taking this approach allows us to get some intuition for why the optimal contract will take the form it does.

First, we will observe that the Investor's break-even constraint must be binding in any optimal contract. To see why, notice that if the constraint were not binding, we could reduce the payment amount w_1 by a little bit and still maintain the break-even constraint. Reducing w_1 makes the incentive-compatibility constraint easier to satisfy, and it increases the Entrepreneur's objective function. This argument tells us that the Entrepreneur will receive all of the surplus the project generates, so her problem is to maximize that surplus.

The second observation is that in any optimal contract, the project is never liquidated following repayment. To see why, suppose $r_0 < r_1 < 1$ so that the project is continued with probability less than one following repayment. Consider an alternative contract in which r_1 is increased to $r_1 + \varepsilon$, for $\varepsilon > 0$ small. Since making this change alone will violate the Investor's breakeven constraint, let us also increase w_1 by εL so that

$$w_1 + \varepsilon L + (1 - r_1 - \varepsilon) L = w_1 + (1 - r_1) L.$$

Under this perturbation, the Investor's breakeven constraint is still satisfied, and the Entrepreneur's incentive-compatibility constraint is satisfied as long as

$$(r_1 + \varepsilon - r_0) p_2 \geq w_1 + \varepsilon L,$$

which is true because $(r_1 - r_0) p_1 \geq w_1$ (or else the original contract did not satisfy IC) and $\varepsilon (p_2 - L) > 0$ since continuing the project is optimal (i.e., $p_2 > L$). If the original contract satisfied IC and IR, then so does this one, but this one also increases the Entrepreneur's objective by $q\left(-\varepsilon L+\varepsilon p_2\right)$, which again is strictly positive, since $p_2 > L$. This perturbation shows that increasing the probability of continuing the project following repayment is good for two reasons: it reduces the probability of inefficient liquidation, and it increases the Entrepreneur's incentives to repay.

Finally, the last step will be to show that the incentive constraint must bind at the optimum. It clearly must be the case that $r_0 < 1$, or else the incentive constraint would be violated. Again, suppose that the incentive constraint was not binding. Then consider a perturbation in which we raise r_0 to $r_0+\varepsilon$, and to maintain the breakeven constraint, we increase w_1 to $w_1+\varepsilon L\left(1-q\right)/q$. If the incentive constraint was not binding, then it will still be satisfied if r_0 is raised by a little bit. Lastly, this perturbation increases the Entrepreneur's payoff by

$$-q\left[\frac{\varepsilon L\left(1-q\right)}{q}\right]+\left(1-q\right)\varepsilon p_2=\left(1-q\right)\left(p_2-L\right)\varepsilon>0.$$

In other words, if the incentive constraint is not binding, it is more efficient for the Entrepreneur to pay the Investor with cash than with an increased probability of liquidation, and since the Entrepreneur captures all the surplus, she will choose to pay in this more efficient way as much as she can.

To summarize, these three perturbations show that any optimal contract in this setting has to satisfy

$$\left(1-r_0^*\right)p_2=w_1^*$$

and

$$qw_1^*+\left(1-q\right)\left(1-r_0^*\right)L=K.$$

This is just two equations in two unknowns, so we can solve for the probability that the project is liquidated following nonpayment:

$$1 - r_0^* = \frac{K}{qp_2 + (1-q)L} > 0.$$

There is a complementarity between the repayment amount and the liquidation probability: if the project requires a lot of capital (i.e., K is large), then the Investor needs to be assured a bigger payment, and in order to assure that bigger payment, the project has to be liquidated with higher probability following nonpayment. If the project has high second-period cash flows (i.e., p_2 is high), then the Entrepreneur loses a lot following nonpayment, so the project does not need to be liquidated with as high of a probability to ensure repayment. Finally, if the liquidation value of the project is high, then the Investor earns more upon liquidation, so he can break even at a lower liquidation probability.

Under the first-best outcome, the project will never be liquidated, and the project will be undertaken as long as the expected cash flows exceed the required capital, or $qp_1 + p_2 > K$. The model features two sources of inefficiencies relative to the first-best outcome. First, in order to assure repayment, the Entrepreneur commits to a contract that with some probability inefficiently liquidates the project.

Second, there is credit rationing: the maximum amount the Entrepreneur can promise the Investor is p_2 in the event that output is high in the first period and L in the event that it is not, so if

$$qp_2 + (1-q)L < K < qp_1 + p_2,$$

the project will be one that should be undertaken but, in equilibrium, will not be undertaken. The liquidation value of the project is related to the collateral value of the assets underlying the project, and there is a literature beginning with Kiyotaki and Moore (1997) that endogenizes the market value of those assets and shows there can be important general equilibrium spillovers across firms.