Economics 2010b Problem Set 5

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Due: Sunday, 3/4 by 11:59pm

Exercise 22 (Adapted from Bolton and Dewtripont, Question 42). Consider the following vertical integration problem: there are two risk-neutral managers, each running an asset a_i, where $i = 1, 2$. Both managers make ex ante investments. Only ex post spot contracts regulating trade are feasible. Ex post trade at price P results in the following payoffs: $R(e_D) - P$ for the downstream manager D and $P - C(e_U)$ for the upstream manager U, where the e_i's denote ex ante investment levels. Investing e_U costs the upstream manager e_U, and investing e_D costs the downstream manager e_D.

If the two managers do not trade with each other, their respective payoffs are

$$r(e_D, \mathcal{A}_D) - P_m \text{ and } P_m - c(e_U, \mathcal{A}_U),$$

where P_m is a market price, and $\mathcal{A}_i$ denotes the collection of assets owned by manager i. In this problem, $\mathcal{A}_i = \emptyset$ under j-integration, $\mathcal{A}_i = \{a_1, a_2\}$ under i-integration, and $\mathcal{A}_i = \{a_i\}$ under nonintegration.

As in the Grossman-Hart-Moore setting, it is assumed that

$$R(e_D) - C(e_U) > r(e_D, \mathcal{A}_1) - c(e_2, \mathcal{A}_2)$$

for all $(e_D, e_U) \in [0, \bar{e}]^2$ and all $\mathcal{A}_i$,

$$R'(e_D) > r'(e_D, \{a_1, a_2\}) \geq r'(e_D, \{a_i\}) \geq r'(e_D, \emptyset) \geq 0,$$

and

$$-C'(e_U) > -c'(e_U, \{a_1, a_2\}) \geq -c'(e_U, \{a_i\}) \geq -c'(e_U, \emptyset) \geq 0.$$

(a) Characterize the first-best allocation of assets and investment levels.

(b) Assuming that the managers split the ex post gains from trade in half, identify conditions on $r'(e_D, \mathcal{A}_i)$ and $c'(e_D, \mathcal{A}_i)$ such that nonintegration is optimal.

Exercise 23. Suppose a downstream buyer D and an upstream seller U meet at date $t = 1$ and trade a widget at date $t = 3$. The value of the widget to the buyer is e_D, and the seller's cost of production is 0. Here, e_D represents an (unverifiable) investment made by the buyer at date $t = 2$. The cost of investment, which is borne entirely by the buyer, is $ce_D^2/2$. No long-term contracts can be written, and there is no discounting.

(a) What is the first-best investment level e_D^{FB}?

(b) Suppose there is a single asset. If the buyer owns it, he has an outside option of λe_D, where $\lambda \in (0, 1)$. If the seller owns it, she has an outside option of v, which is independent of and smaller than e_D. (Imagine that the seller can sell the asset for v in the outside market, and the minimal investment e_D is bigger than v.) Assume that the buyer and seller divide the ex post gains from trade 50 : 50 (Nash bargaining).

Compute the buyer's investment for the case where the buyer owns the asset and for the case where the seller owns the asset.

(c) Now assume a different bargaining game at date $t = 3$. If both parties have outside options that are valued below $e_D/2$, the parties split the surplus, giving $e_D/2$ to each party. If one of the parties has an outside option that gives $r > e_D/2$, then the party gets r and the other party gets the remainder $e_D - r$. Supposing that $\lambda > 1/2$, compute the buyer's investment when the buyer owns the asset. Compare this with the outcome when the seller owns the asset, distinguishing between the situations where v is high and v is low. Note: for this part, assume that, under S-ownership, B's outside option is $\bar{w} < -v$, making it irrelevant.

Long Hint: this part is a bit complicated due to the non-standard bargaining game, but it is illustrative of how the bargaining structure affects investment incentives (and it makes Nash bargaining look very nice in comparison). This hint is meant to guide you through the problem.

- Under seller ownership, the bargaining game is such that the buyer chooses e_D to

$$\max_{e_D} \left\{ \min \left\{ e_D - v, \frac{e_D}{2} \right\} - \frac{c}{2} e_D^2 \right\}.$$

- Break it up into cases:
  - If $e_D - v < e_D/2$, then what is the buyer's optimal choice of e_D? Plug back in to check that the condition holds.
  - If $e_D - v > e_D/2$, then what is the buyer's optimal choice of e_D? Plug back in to check that the condition holds—what happens if it does not?
- Write the buyer's optimal choice of e_D as a step function with arguments v and c.